MATHEMATICS

MINIMUM LEVEL LEARNING MATERIAL AND SAMPLE PAPERS

CLASS – X

2018 – 19

MINIMUM LEVEL DAILY REVISION SYLLABUS
FOR REMEDIAL STUDENTS
MATHEMATICS: CLASS X

S. NO.	CHAPTER/TOPIC	MARKS COVERED AS PER LATEST CBSE SAMPLE PAPERS
1	Real Numbers – Full Chapter	6
2	Polynomials – Full Chapter	4
3	Arithmetic Progression – Full Chapter	7
4	Triangles Theorem	4
5	Circles – Full Chapter	3
6	Construction – Full Chapter	4
7	Areas related to Circles	3
8	Statistics – Full Chapter	7
9	Probability – Full Chapter	4
Total Marks		42

All Remedial Students have to complete the above chapters/topics thoroughly with 100% perfection and then they can also concentrate the below topics for Board Exam:

*Linear Equation in two variables **– Graph Questions, Comparing the ratios of coefficients based questions.**

*Quadratic Equations **– imp questions**

*Triangles **– 1 mark imp questions**

*Coordinate Geometry **– imp questions**

*Trigonometry **– imp questions**

*Surface Areas and Volumes **– imp questions**

INDEX OF MINIMUM LEVEL LEARNING STUDY MATERIAL

CLASS X : MATHEMATICS

CHAPTER – 1
REAL NUMBERS

EUCLID'S DIVISION LEMMA

Given positive integers a and b, there exist unique integers q and r satisfying a = bq + r, where $0 \le r < b$.

Here we call 'a' as dividend, 'b' as divisor, 'q' as quotient and 'r' as remainder.

$\therefore$ Dividend = (Divisor x Quotient) + Remainder

If in Euclid's lemma r = 0 then b would be HCF of 'a' and 'b'.

IMPORTANT QUESTIONS

Show that any positive even integer is of the form 6q, or 6q + 2, or 6q + 4, where q is some integer.

Solution: Let x be any positive integer such that x > 6. Then, by Euclid's algorithm,
x = 6q + r for some integer q ≥ 0 and 0 ≤ r < 6.
Therefore, x = 6q or 6q + 1 or 6q + 2 or 6q + 3 or 6q + 4 or 6q + 5
Now, 6q is an even integer being a multiple of 2.
We know that the sum of two even integers are always even integers.
Therefore, 6q + 2 and 6q + 4 are even integers
Hence any positive even integer is of the form 6q, or 6q + 2, or 6q + 4, where q is some integer.

Questions for practice

1. Show that any positive even integer is of the form 4q, or 4q + 2, where q is some integer.
2. Show that any positive odd integer is of the form 4q + 1, or 4q + 3, where q is some integer.
3. Show that any positive odd integer is of the form 6q + 1, or 6q + 3, or 6q + 5, where q is some integer.
4. Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.
5. Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8.
6. Use Euclid's division lemma to show that the square of an odd positive integer can be of the form 6q + 1 or 6q + 3 for some integer q.
7. Use Euclid's division lemma to prove that one and only one out of n, n + 2 and n + 4 is divisible by 3, where n is any positive integer.
8. Use Euclid's division lemma to prove that one of any three consecutive positive integers must be divisible by 3.
9. For any positive integer n, use Euclid's division lemma to prove that $n^3 - n$ is divisible by 6.
10. Use Euclid's division lemma to show that one and only one out of n, n + 4, n + 8, n + 12 and n + 16 is divisible by 5, where n is any positive integer.

EUCLID'S DIVISION ALGORITHM

Euclid's division algorithm is a technique to compute the Highest Common Factor (HCF) of two given positive integers. Recall that the HCF of two positive integers *a* and *b* is the largest positive integer *d* that divides both *a* and *b*.

To obtain the HCF of two positive integers, say c and d, with c > d, follow the steps below:

Step 1 : Apply Euclid's division lemma, to *c* and *d*. So, we find whole numbers, *q* and *r* such that *c* = *dq* + *r*, $0 \le r < d$.

Step 2 : If *r* = 0, *d* is the HCF of *c* and *d*. If $r \ne 0$ apply the division lemma to *d* and *r*.

Step 3 : Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.

This algorithm works because HCF (c, d) = HCF (d, r) where the symbol HCF (c, d) denotes the HCF of c and d, etc.

IMPORTANT QUESTIONS

Use Euclid's division algorithm to find the HCF of 867 and 255

Solution: Since 867 > 255, we apply the division lemma to 867 and 255 to obtain
$867 = 255 \times 3 + 102$
Since remainder $102 \neq 0$, we apply the division lemma to 255 and 102 to obtain
$255 = 102 \times 2 + 51$
We consider the new divisor 102 and new remainder 51, and apply the division lemma to obtain
$102 = 51 \times 2 + 0$
Since the remainder is zero, the process stops.
Since the divisor at this stage is 51,
Therefore, HCF of 867 and 255 is 51.

Questions for practice

1. Use Euclid's algorithm to find the HCF of 4052 and 12576.
2. Use Euclid's division algorithm to find the HCF of 135 and 225.
3. Use Euclid's division algorithm to find the HCF of 196 and 38220.
4. Use Euclid's division algorithm to find the HCF of 455 and 42.
5. Using Euclid's division algorithm, find which of the following pairs of numbers are co-prime: (i) 231, 396 (ii) 847, 2160
6. If the HCF of 65 and 117 is expressible in the form 65m – 117, then find the value of m.

The Fundamental Theorem of Arithmetic

Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.

The prime factorisation of a natural number is unique, except for the order of its factors.

❖ Property of HCF and LCM of two positive integers 'a' and 'b':
- $HCF(a,b) \times LCM(a,b) = a \times b$
- $LCM(a,b) = \dfrac{a \times b}{HCF(a,b)}$
- $HCF(a,b) = \dfrac{a \times b}{LCM(a,b)}$

PRIME FACTORISATION METHOD TO FIND HCF AND LCM

HCF(a, b) = Product of the smallest power of each common prime factor in the numbers.
LCM(a, b) = Product of the greatest power of each prime factor, involved in the numbers.

IMPORTANT QUESTIONS

Find the LCM and HCF of 510 and 92 and verify that LCM × HCF = product of the two numbers

Solution: 510 = 2 x 3 x 5 x 17
92 = 2 x 2 x 23 = 2^2 x 23
HCF = 2
LCM = 2^2 x 3 x 5 x 17 x 23 = 23460
Product of two numbers = 510 x 92 = 46920

HCF x LCM = 2 x 23460 = 46920
Hence, product of two numbers = HCF × LCM

Questions for practice

1. Find the HCF and LCM of 6, 72 and 120, using the prime factorisation method.
2. Find the HCF of 96 and 404 by the prime factorisation method. Hence, find their LCM.
3. Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers: (i) 26 and 91 (ii) 336 and 54
4. Find the LCM and HCF of the following integers by applying the prime factorisation method: (i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25
5. Explain why 3 × 5 × 7 + 7 is a composite number.
6. Can the number 6^n, n being a natural number, end with the digit 5? Give reasons.
7. Can the number 4^n, n being a natural number, end with the digit 0? Give reasons.
8. Given that HCF (306, 657) = 9, find LCM (306, 657).
9. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$; x, y are prime numbers, then find the HCF (a, b).
10. If two positive integers p and q can be expressed as $p = ab^2$ and $q = a^3b$; a, b being prime numbers, then find the LCM (p, q).

IRRATIONALITY OF NUMBERS

IMPORTANT QUESTIONS

Prove that $\sqrt{5}$ is an irrational number.

Solution: Let $\sqrt{5}$ is a rational number then we have

$\sqrt{5} = \frac{p}{q}$, where p and q are co-primes.

$\Rightarrow p = \sqrt{5}q$

Squaring both sides, we get

$p^2 = 5q^2$

$\Rightarrow$ p^2 is divisible by 5

$\Rightarrow$ p is also divisible by 5

So, assume p = 5m where m is any integer.

Squaring both sides, we get $p^2 = 25m^2$

But $p^2 = 5q^2$

Therefore, $5q^2 = 25m^2$

$\Rightarrow q^2 = 5m^2$

$\Rightarrow$ q^2 is divisible by 5

$\Rightarrow$ q is also divisible by 5

From above we conclude that p and q has one common factor i.e. 5 which contradicts that p and q are co-primes.

Therefore our assumption is wrong.

Hence, $\sqrt{5}$ is an irrational number.

Questions for practice

1. Prove that $\sqrt{2}$ is an irrational number.
2. Prove that $\sqrt{3}$ is an irrational number.
3. Prove that $2 + 5\sqrt{3}$ is an irrational number.
4. Prove that $3 - 2\sqrt{5}$ is an irrational number.
5. Prove that $\sqrt{2} + \sqrt{3}$ is an irrational number.

RATIONAL NUMBERS AND THEIR DECIMAL EXPANSIONS

Let x = $\frac{p}{q}$ be a rational number, such that the prime factorisation of q is of the form $2^m.5^n$, where m, n are non-negative integers. Then x has a decimal expansion which terminates.

Let x = $\frac{p}{q}$ be a rational number, such that the prime factorisation of q is not of the form $2^m.5^n$, where m, n are non-negative integers. Then x has a decimal expansion which is non-terminating repeating (recurring).

IMPORTANT QUESTIONS

Without actually performing the long division, state whether the rational numbers $\frac{987}{10500}$ will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

Solution: Given rational number $\frac{987}{10500}$ is not in the simplest form. Dividing numerator and denominator by 21 we get $\frac{987}{10500} = \frac{987 \div 21}{10500 \div 21} = \frac{47}{500}$ which is in the form of $\frac{p}{q}$

Now q = 500 = 2^2 x 5^3 which is in the form of $2^m.5^n$, where m, n are non-negative integers. Therefore the given rational number has terminating decimal expansion.

Questions for practice

Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

$(i)\frac{13}{3125}\ (ii)\frac{129}{2^2 5^7 7^5}\ (iii)\frac{77}{210}\ (iv)\frac{14587}{1250}\ (v)\frac{833}{2^2 5^5 7^2}$

CHAPTER – 2
POLYNOMIALS

QUADRATIC POLYNOMIAL

Relationship between zeroes and coefficients

General form of Quadratic polynomial: $ax^2 + bx + c,\ a \neq 0$

Sum of zeroes $(\alpha + \beta) = -\dfrac{\text{Coefficient of x}}{\text{Coefficient of } x^2} = -\dfrac{b}{a}$

Product of zeroes $(\alpha\beta) = \dfrac{\text{Constant term}}{\text{Coefficient of } x^2} = \dfrac{c}{a}$

IMPORTANT QUESTIONS

Find a quadratic polynomial, the sum and product of whose zeroes are – 3 and 2, respectively.

Solution: Here, $\alpha + \beta = -3$ and $\alpha\beta = 2$

We know that quadratic polynomial is given by $p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - (-3)x + 2 = x^2 + 3x + 2$

Hence, required quadratic polynomial is $x^2 + 3x + 2$

Find a quadratic polynomial, whose zeroes are – 3 and 2.

Solution: Here, $\alpha = -3$ and $\beta = 2$.

Now, $\alpha + \beta = -3 + 2 = -1$ and $\alpha\beta = (-3)(2) = -6$

We know that quadratic polynomial is given by $p(x) = x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - (-1)x + (-6) = x^2 + x - 6$

Hence, required quadratic polynomial is $x^2 + x - 6$

Find the zeroes of the quadratic polynomial $x^2 - 2x - 8$ and verify the relationship between the zeroes and the coefficients.

Solution: Here, $p(x) = x^2 - 2x - 8 = 0$

$x^2 - 4x + 2x - 8 = 0 \Rightarrow x(x-4) + 2(x-4) = 0 \Rightarrow (x-4)(x+2) = 0$

$\Rightarrow x = 4, -2$

Now, $a = 1,\ b = -2,\ c = -8,\ \alpha = 4,\ \beta = -2$

Sum of zeroes, $\alpha + \beta = 4 + (-2) = 2$ and $\dfrac{-b}{a} = \dfrac{-(-2)}{1} = 2 \qquad \therefore \alpha + \beta = \dfrac{-b}{a}$

Product of zeroes, $\alpha\beta = 4(-2) = -8$ and $\dfrac{c}{a} = \dfrac{-8}{1} = -8 \qquad \therefore \alpha\beta = \dfrac{c}{a}$.

Hence verified.

Questions for practice

1. Find a quadratic polynomial, the sum and product of whose zeroes are – 5 and 3, respectively.
2. Find a quadratic polynomial, whose zeroes are – 4 and 1, respectively.
3. Find the zeroes of the quadratic polynomial $x^2 + 7x + 10$, and verify the relationship between the zeroes and the coefficients.
4. Find the zeroes of the polynomial $x^2 - 3$ and verify the relationship between the zeroes and the coefficients.
5. Find the zeroes of the quadratic polynomial $6x^2 - 3 - 7x$ and verify the relationship between the zeroes and the coefficients.
6. Find the zeroes of the quadratic polynomial $3x^2 - x - 4$ and verify the relationship between the zeroes and the coefficients.
7. Find the zeroes of the quadratic polynomial $4x^2 - 4x + 1$ and verify the relationship between the zeroes and the coefficients.

DIVISION ALGORITHM FOR POLYNOMIALS

If $p(x)$ and $g(x)$ are any two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$
such that $p(x) = g(x) \times q(x) + r(x)$, where $r(x) = 0$ or degree of $r(x)$ < degree of $g(x)$.

- ❖ If r(x) = 0, then g(x) is a factor of p(x).
- ❖ Dividend = Divisor × Quotient + Remainder

IMPORTANT QUESTIONS

Divide $3x^2 - x^3 - 3x + 5$ by $x - 1 - x^2$, and verify the division algorithm.

Solution:

$$
\begin{array}{r|l}
 & x - 2 \\
\hline
-x^2 + x - 1 & -x^3 + 3x^2 - 3x + 5 \\
 & -x^3 + x^2 - x \\
 & \quad + \quad - \quad + \\
\hline
 & \quad 2x^2 - 2x + 5 \\
 & \quad 2x^2 - 2x + 2 \\
 & \quad - \\
\hline
 & \qquad\qquad 3
\end{array}
$$

So, quotient = x – 2, remainder = 3.
Now, Divisor × Quotient + Remainder
$= (-x^2 + x - 1)(x - 2) + 3$
$= -x^3 + x^2 - x + 2x^2 - 2x + 2 + 3$
$= -x^3 + 3x^2 - 3x + 5$
= Dividend
Hence, the division algorithm is verified.

Questions for Practice

1. Divide $3x^3 + x^2 + 2x + 5$ by $1 + 2x + x^2$.

2. Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following :
   (i) $p(x) = x^3 - 3x^2 + 5x - 3$, $g(x) = x^2 - 2$
   (ii) $p(x) = x^4 - 3x^2 + 4x + 5$, $g(x) = x^2 + 1 - x$
   (iii) $p(x) = x^4 - 5x + 6$, $g(x) = 2 - x^2$

3. Find all the zeroes of $2x^4 - 3x^3 - 3x^2 + 6x - 2$, if you know that two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$

4. Obtain all other zeroes of $3x^4 + 6x^3 - 2x2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

5. On dividing $x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were x – 2 and –2x + 4, respectively. Find g(x).

6. If the remainder on division of $x^3 + 2x^2 + kx + 3$ by x – 3 is 21, find the quotient and the value of k. Hence, find the zeroes of the cubic polynomial $x^3 + 2x^2 + kx - 18$.

7. Find k so that $x^2 + 2x + k$ is a factor of $2x^4 + x^3 - 14x^2 + 5x + 6$. Also find all the zeroes of the two polynomials.

CHAPTER – 5
ARITHMETIC PROGRESSION

<u>*n*th Term of an ARITHMETIC PROGRESSION (AP)</u>

*n*th term a_n of the AP with first term a and common difference d is given by

$$\boxed{a_n = a + (n-1)\,d.}$$

IMPORTANT QUESTIONS

Find the 15th term of the 21, 24, 27, . . .

Solution: Here, a = 21, d = 24 – 21 = 3

We know that $a_n = a + (n - 1)d$

So, $a_{15} = a + 14d = 21 + 14(3) = 21 + 42 = 63$

Which term of the AP : 3, 9, 15, 21, . . . , is 99?

Solution: Here, a = 3, d = 9 – 3 = 6

We know that $a_n = a + (n - 1)d$

Let an = 99 $\Rightarrow$ a + (n – 1)d = 99

$\Rightarrow 3 + (n - 1)6 = 99 \Rightarrow (n - 1)6 = 99 - 3 = 96$

$\Rightarrow n - 1 = \frac{96}{6} = 16 \quad \Rightarrow n = 16 + 1 = 17$

Hence, 17th term of the given AP is 99

Determine the AP whose 3rd term is 5 and the 7th term is 9.

Solution: We have $a_3 = a + (3 - 1)\,d = a + 2d = 5$ .... (1)

and $a_7 = a + (7 - 1)\,d = a + 6d = 9$ .... (2)

Solving the pair of linear equations (1) and (2), we get a = 3, d = 1

Hence, the required AP is 3, 4, 5, 6, 7, . . .

Questions for practice

1. Find the 10th term of the AP : 2, 7, 12, . . .
2. Which term of the AP : 21, 18, 15, . . . is – 81?
3. Which term of the AP : 3, 8, 13, 18, . . . ,is 78?
4. How many two-digit numbers are divisible by 3?
5. How many three-digit numbers are divisible by 7?
6. How many multiples of 4 lie between 10 and 250?
7. Find the 31st term of an AP whose 11th term is 38 and the 16th term is 73.
8. An AP consists of 50 terms of which 3rd term is 12 and the last term is 106. Find the 29th term.
9. If the 3rd and the 9th terms of an AP are 4 and – 8 respectively, which term of this AP is zero?
10. Which term of the AP : 3, 15, 27, 39, . . . will be 132 more than its 54th term?
11. Determine the AP whose third term is 16 and the 7th term exceeds the 5th term by 12.
12. The sum of 4th term and 8th term of an AP is 24 and the sum of 6th and 10th terms is 44. Find the AP.
13. The sum of 5th term and 9th term of an AP is 72 and the sum of 7th and 12th terms is 97. Find the AP.
14. If the numbers n – 2, 4n – 1 and 5n + 2 are in AP, find the value of n.
15. Find the value of the middle most term (s) of the AP : –11, –7, –3,..., 49.
16. The sum of the first three terms of an AP is 33. If the product of the first and the third term exceeds the second term by 29, find the AP.
17. The sum of the 5th and the 7th terms of an AP is 52 and the 10th term is 46. Find the AP.

18. Find the 20th term of the AP whose 7th term is 24 less than the 11th term, first term being 12.
19. If the 9th term of an AP is zero, prove that its 29th term is twice its 19th term.
20. Which term of the AP: 53, 48, 43,... is the first negative term?
21. A sum of Rs 1000 is invested at 8% simple interest per year. Calculate the interest at the end of each year. Do these interests form an AP? If so, find the interest at the end of 30 years making use of this fact.
22. In a flower bed, there are 23 rose plants in the first row, 21 in the second, 19 in the third, and so on. There are 5 rose plants in the last row. How many rows are there in the flower bed?

***n*th Term from the end of an ARITHMETIC PROGRESSION (AP)**

Let the last term of an AP be '*l*' and the common difference of an AP is 'd' then the nth term from the end of an AP is given by

$$\boxed{l_n = l - (n - 1)\, d.}$$

IMPORTANT QUESTIONS

Find the 11th term from the last term (towards the first term) of the AP : 10, 7, 4, . . ., – 62.

Solution : Here, a = 10, d = 7 – 10 = – 3, l = – 62,

We know that nth term from the last is given by $l_n = l - (n - 1)\, d$.

$\therefore l_{11} = l - 10d = -62 - 10(-3) = -62 + 30 = -32$

Questions for practice

1. Find the 20th term from the last term of the AP : 3, 8, 13, . . ., 253.
2. Find the 10th term from the last term of the AP : 4, 9 , 14, . . ., 254.
3. Find the 6th term from the end of the AP 17, 14, 11, …… (–40).
4. Find the 8th term from the end of the AP 7, 10, 13, …… 184.
5. Find the 10th term from the last term of the AP : 8, 10, 12, . . ., 126.

Sum of First *n* Terms of an ARITHMETIC PROGRESSION (AP)

The sum of the first *n* terms of an AP is given by

$$\boxed{S_n = \frac{n}{2}[2a + (n-1)d]}$$

where a = first term, d = common difference and n = number of terms.

or

$$\boxed{S_n = \frac{n}{2}[a + l]}$$

where *l* = last term

IMPORTANT QUESTIONS

Find the sum of the first 22 terms of the AP : 8, 3, –2, . . .

Solution : Here, a = 8, d = 3 – 8 = –5, n = 22.

We know that $S = \frac{n}{2}[2a + (n-1)d]$

$\therefore S = \frac{22}{2}[16 + (22-1)\times(-5)] = 11(16 - 105) = 11(-89) = -979$

So, the sum of the first 22 terms of the AP is – 979.

Questions for practice

1. If the sum of the first 14 terms of an AP is 1050 and its first term is 10, find the 20th term.
2. How many terms of the AP : 24, 21, 18, . . . must be taken so that their sum is 78?

3. How many terms of the AP : 9, 17, 25, . . . must be taken to give a sum of 636?

4. Find the sum of first 24 terms of the list of numbers whose nth term is given by an = 3 + 2n

5. Find the sum of the first 40 positive integers divisible by 6.

6. Find the sum of the first 15 multiples of 8.

7. Find the sum of the odd numbers between 0 and 50.

8. Find the sum of first 22 terms of an AP in which d = 7 and 22nd term is 149.

9. Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

10. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

11. If $a_n = 3 – 4n$, show that $a_1, a_2, a_3, ...$ form an AP. Also find S_{20} .

12. In an AP, if $S_n = n (4n + 1)$, find the AP.

13. In an AP, if $S_n = 3n^2 + 5n$ and $a_k = 164$, find the value of k.

14. If S_n denotes the sum of first n terms of an AP, prove that $S_{12} = 3(S_8 – S_4)$

15. Find the sum of first 17 terms of an AP whose 4th and 9th terms are –15 and –30 respectively.

16. If sum of first 6 terms of an AP is 36 and that of the first 16 terms is 256, find the sum of first 10 terms.

17. Find the sum of all the 11 terms of an AP whose middle most term is 30.

18. Find the sum of last ten terms of the AP: 8, 10, 12,---, 126.

19. How many terms of the AP: –15, –13, –11,--- are needed to make the sum –55? Explain the reason for double answer.

20. The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x.

21. A manufacturer of TV sets produced 600 sets in the third year and 700 sets in the seventh year. Assuming that the production increases uniformly by a fixed number every year, find : (i) the production in the 1st year (ii) the production in the 10th year (iii) the total production in first 7 years

22. 200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how may rows are the 200 logs placed and how many logs are in the top row?

23. A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs 200 for the first day, Rs 250 for the second day, Rs 300 for the third day, etc., the penalty for each succeeding day being Rs 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days?

24. A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 20 less than its preceding prize, find the value of each of the prizes.

25. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of Class I will plant 1 tree, a section of Class II will plant 2 trees and so on till Class XII. There are three sections of each class. How many trees will be planted by the students?

26. A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm, ... What is the total length of such a spiral made up of thirteen consecutive semicircles?

27. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line. A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

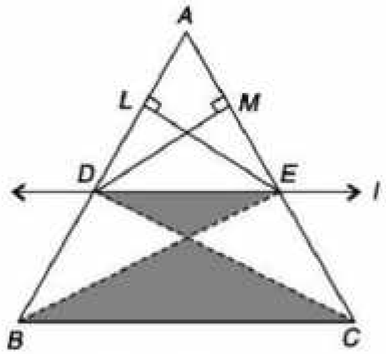
CHAPTER – 6
TRIANGLES

IMPORTANT THEOREMS

BASIC PROPORTIONALITY THEOREM OR THALES THEOREM

If a straight line is drawn parallel to one side of a triangle intersecting the other two sides, then it divides the two sides in the same ratio.

GIVEN: A ΔABC and line 'l' parallel to BC intersect AB at D and AC at E.



TO PROVE : $$\frac{AD}{DB} = \frac{AE}{EC}$$

CONSTRUCTION : Join BE and CD. Draw $EL \perp$ to AB and $DM \perp AC$.

PROOF: We know that areas of the triangles on the same base and between same parallel lines are equal, hence we have :

$$\text{area } (\Delta BDE) = \text{area } (\Delta CDE) \quad \text{...(i)}$$

Now, we have

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta BDE} = \frac{\frac{1}{2} \times AD \times EL}{\frac{1}{2} \times DB \times EL} = \frac{AD}{DB} \quad \text{...(ii)}$$

Again, we have

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta CDE} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \quad \text{...(iii)}$$

Put value form (i) in (ii), we have

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta CDE} = \frac{AD}{DB} \quad \text{...(iv)}$$

On comparing equation (ii) and (iii), we get

$$\frac{AD}{DB} = \frac{AE}{EC}$$ **Hence Proved.**

COROLLARY :

(i) $\frac{AB}{DB} = \frac{AC}{EC}$

(ii) $\frac{DB}{AD} = \frac{EC}{AE}$

(iii) $\frac{AB}{AD} = \frac{AC}{AE}$

(iv) $\frac{DB}{AB} = \frac{EC}{AC}$

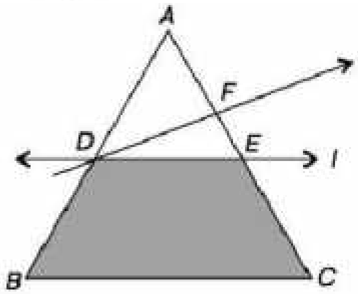
(v) $\frac{AD}{AB} = \frac{AE}{AC}$

CONVERSE OF BASIC PROPORTIONALITY THEOREM
(CONVERSE OF THALES THEOREM)

If a straight line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

GIVEN : A Δ ABC and line 'l' intersecting the sides AB at D and AC at E such that :

$$\frac{AD}{DB} = \frac{AE}{EC}$$



TO PROVE : $l \parallel BC$.

PROOF : Let us suppose that the line l is not parallel to BC.

Then through D, there must be any other line which must be parallel to BC.

Let $DF \parallel BC$, such that $E \neq F$.

Since, $DF \parallel BC$ (by supposition)

$$\frac{AD}{DB} = \frac{AF}{FC} \quad \text{...(i) (Basic Proportionality Theorem)}$$

$$\frac{AD}{DB} = \frac{AE}{EC} \quad \text{...(ii) (Given)}$$

Comparing (i) and (ii), we get

$$\frac{AF}{FC} = \frac{AE}{EC}$$

Adding 1 to both sides, we get

$$\frac{AF}{FC}+1 = \frac{AE}{EC}+1$$

$$\Rightarrow \quad \frac{AF+FC}{FC} = \frac{AE+EC}{EC}$$

$$\Rightarrow \quad \frac{AC}{FC} = \frac{AC}{EC}$$

$$\Rightarrow \quad \frac{1}{FC} = \frac{1}{EC}$$

$$\Rightarrow \quad FC = EC$$

This shows that E and F must coincide, but it contradicts our supposition that $E \neq F$ and $DF \parallel BC$.

Hence, there is one and only line, $DE \parallel BC$, *i.e.*

$$\boxed{l \parallel BC}$$

Hence Proved.

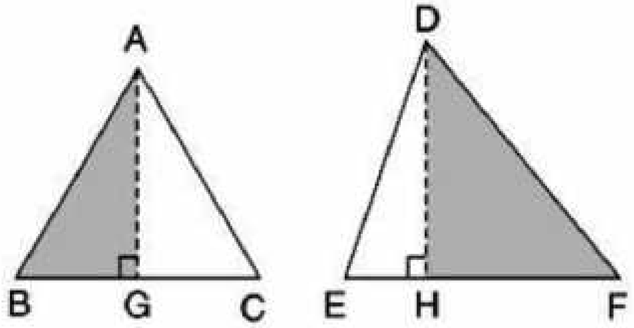
AREAS OF SIMILAR TRIANGLES THEOREM

The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

GIVEN : $\Delta ABC \sim \Delta DEF$

TO PROVE : $\frac{\text{Area of } (\Delta ABC)}{\text{Area of } (\Delta DEF)} = \frac{BC^2}{EF^2} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2}$

CONSTRUCTION : Draw $AG \perp BC$ and $DH \perp EF$.



PROOF :

$$\frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DEF)} = \frac{\frac{1}{2}\times BC\times AG}{\frac{1}{2}\times EF\times DH} = \frac{BC}{EF}\times\frac{AG}{DH} \qquad \text{...(i)}$$

(area of Δ = ½ × base × height)

Now in triangle ABG and DEH, we have

$\angle B = \angle E$ (since, $\Delta ABC \sim \Delta DEF$)

$\angle AGB = \angle DHE$ (each 90°)

Therefore, $\Delta ABG \sim \Delta DEH$ (by AA criterion)

Hence, $\dfrac{AB}{DE} = \dfrac{AG}{DH}$ ...(ii) (Using property of similar triangles)

$\Rightarrow$ $\dfrac{AB}{DE} = \dfrac{BC}{EF}$ ...(iii) (since, $\Delta ABC \sim \Delta DEF$)

Comparing (ii) and (iii), we get

$$\frac{AG}{DH} = \frac{BC}{EF} \qquad \text{...(iv)}$$

Using (i) and (iv), we get

$$\frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta DEF)} = \frac{BC}{EF}\times\frac{BC}{EF} = \frac{BC^2}{EF^2} \qquad \text{...(v)}$$

$$\frac{BC}{EF} = \frac{AB}{DE} = \frac{AC}{DF} \qquad \text{...(vi) (since, } \Delta ABC \sim \Delta DEF)$$

Using (v) and (vi), we get

$$\boxed{\frac{\text{Area of }(\Delta ABC)}{\text{Area of }(\Delta DEF)} = \frac{BC^2}{EF^2} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2}}$$

Hence Proved.

PYTHAGORAS THEOREM

In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

GIVEN : ΔABC is right angled at B.

TO PROVE : $AC^2 = AB^2 + BC^2$.

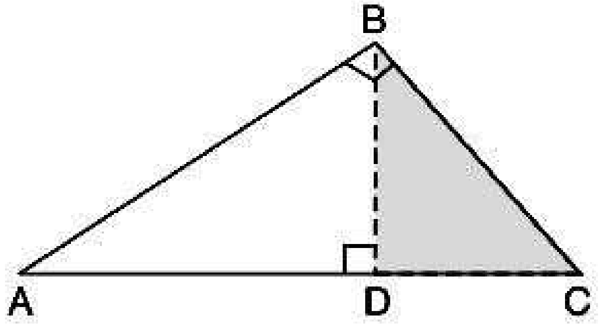
CONSTRUCTION : Draw $BD \perp AC$.

PROOF : Taking ΔADB and ΔABC

$\angle B = \angle ADB$ (each 90°)

$\angle A = \angle A$ (common)

Therefore, $\Delta ADB \sim \Delta ABC$ (by AA criterion)



Hence, $\dfrac{AD}{AB} = \dfrac{AB}{AC}$

$\Rightarrow$ $AB^2 = AD \times AC$ ...(i)

Now taking $\Delta\, CDB$ and $\Delta\, CBA$

$\angle B = \angle BDC$ (each 90°)

$\angle C = \angle C$ (common)

Therefore, $\Delta\, CDB \sim \Delta\, CBA$ (by AA criterion)

Hence, $\dfrac{CD}{BC} = \dfrac{BC}{AC}$

$\Rightarrow$ $BC^2 = CD \times AC$ ...(ii)

Adding (i) and (ii), we get

$\Rightarrow$ $AB^2 + BC^2 = AD \times AC + CD \times AC$

$\Rightarrow$ $AB^2 + BC^2 = AC \times (AD + CD)$

$\Rightarrow$ $AB^2 + BC^2 = AC \times AC$

$\boxed{AC^2 = AB^2 - BC^2}$ **Hence Proved.**

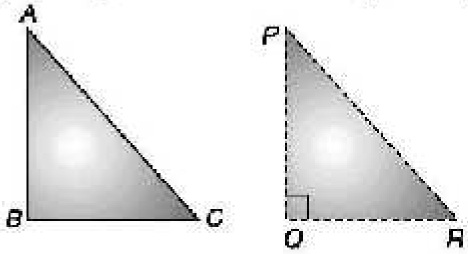
Converse of Pythagoras theorem

In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

GIVEN : $AC^2 = AB^2 + BC^2$. ...(i)

TO PROVE : $\Delta\, ABC$ is right angled at B.

CONSTRUCTION : Draw right $\Delta\, PQR$ such that $AB = PQ$, $BC = QR$ and $\angle Q = 90°$.



PROOF : Using Pythagoras theorem in $\Delta\, PQR$, we get

$PR^2 = PQ^2 + QR^2$ ...(ii)

By construction, $AB = PQ$

$BC = QR$, substituting these values in (ii), we get

$PR^2 = AB^2 + BC^2$ ...(iii)

Comparing (i) and (iii), we get

$AC^2 = PR^2$

$\Rightarrow$ $AC = PR$ ...(iv)

In ΔABC and ΔPQR

$AB = PQ$ (by construction)

$BC = QR$ (by construction)

$AC = PR$ (proved above in (iv))

$\Rightarrow$ $\Delta ABC \cong \Delta PQR$ (by SSS congruence rule)

$\Rightarrow$ $\angle B = \angle Q$ (by cpct)

But $\angle Q = 90^\circ$ (by construction)

Hence, $\angle B = 90^\circ$

$\boxed{\Delta ABC \text{ is right angled at } B.}$

Hence Proved.

CHAPTER – 10
CIRCLES

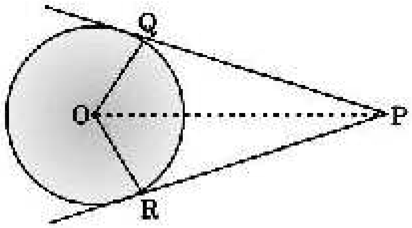
THEOREMS

1) The tangent to a circle is perpendicular to the radius through the point of contact.

2) The lengths of tangents drawn from an external point to a circle are equal.

Given : A circle $C(O, r)$ and two tangents say PQ and PR from an external point P.

To prove : $PQ = PR$.



Construction : Join OQ, OR and OP.

Proof : In ΔOQP and ΔORP

$OQ = OR$ (radii of the same circle)

$OP = OP$ (Common)

$\angle Q = \angle R$ = each 90° (The tangent at any point of a circle is perpendicular to the radius through the point of contact)

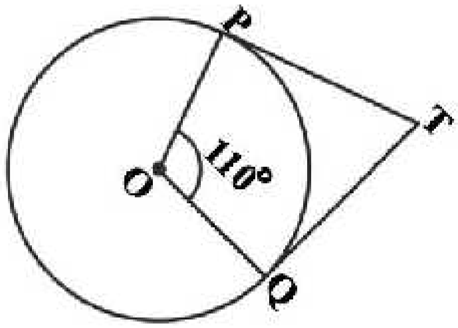
Hence $\Delta OQP \cong \Delta ORP$ (By RHS Criterion)

$\therefore$ $PQ = PR$

(By CPCT) **Hence Proved.**

IMPORTANT QUESTIONS

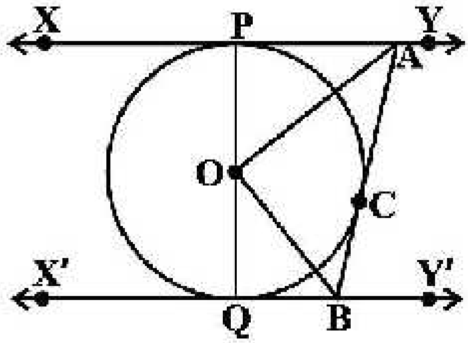
1. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. Find the radius of the circle
2. In the below figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110°$, then find $\angle PTQ$.



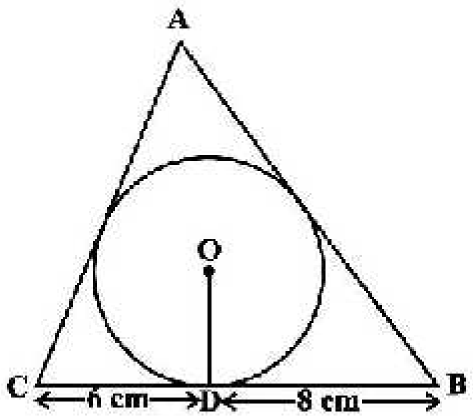
3. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle of 80°, then find $\angle POA$
4. The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.
5. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.
6. A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC
7. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.
8. Prove that the parallelogram circumscribing a circle is a rhombus.
9. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

10. Prove that in two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.

11. XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X′Y′ at B. Prove that $\angle AOB = 90°$.



12. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively. Find the sides AB and AC.



13. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.

14. PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length TP.

15. Two tangents PQ and PR are drawn from an external point to a circle with centre O. Prove that QORP is a cyclic quadrilateral.

16. If from an external point B of a circle with centre O, two tangents BC and BD are drawn such that $\angle DBC = 120°$, prove that BC + BD = BO, i.e., BO = 2BC.

17. Prove that the tangents drawn at the ends of a chord of a circle make equal angles with the chord.

18. Prove that a diameter AB of a circle bisects all those chords which are parallel to the tangent at the point A.

19. From an external point P, two tangents, PA and PB are drawn to a circle with centre O. At one point E on the circle tangent is drawn which intersects PA and PB at C and D, respectively. If PA = 10 cm, find the the perimeter of the triangle PCD.

20. In a right triangle ABC in which $\angle B = 90°$, a circle is drawn with AB as diameter intersecting the hypotenuse AC and P. Prove that the tangent to the circle at P bisects BC.

CHAPTER – 11
CONSTRUCTIONS

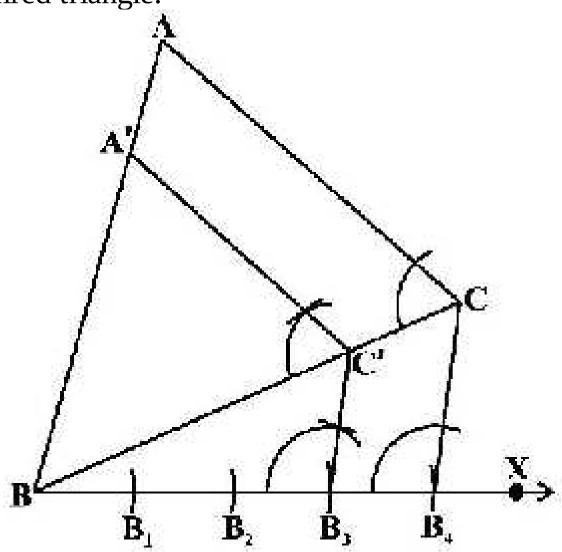
CONSTRUCTION OF SIMILAR TRIANGLE

Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{3}{4}$ of the corresponding sides of the triangle ABC (i.e., of scale factor $\frac{3}{4}$).

<u>Steps of Construction :</u>

- ☞ Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.
- ☞ Locate 4 (the greater of 3 and 4 in $\frac{3}{4}$) points B_1, B_2, B_3 and B_4 on BX so that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.
- ☞ Join B_4C and draw a line through B3 (the 3rd point, 3 being smaller of 3 and 4 in $\frac{3}{4}$) parallel to B_4C to intersect BC at C′.
- ☞ Draw a line through C′ parallel to the line CA to intersect BA at A′ (see below figure).

Then, Δ A′BC′ is the required triangle.



Construct a triangle similar to a given triangle ABC with its sides equal to $\frac{5}{3}$ of the corresponding sides of the triangle ABC (i.e., of scale factor $\frac{5}{3}$).

<u>Steps of Construction</u> :

- ➢ Draw any ray BX making an acute angle with BC on the side opposite to the vertex A.
- ➢ Locate 5 points (the greater of 5 and 3 in $\frac{5}{3}$) B1, B2, B3, B4 and B5 on BX so that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$.
- ➢ Join B_3(the 3rd point, 3 being smaller of 3 and 5 in $\frac{5}{3}$) to C and draw a line through B_5 parallel to B_3C, intersecting the extended line segment BC at C′.
- ➢ Draw a line through C′ parallel to CA intersecting the extended line segment BA at A′ (see the below figure).
  Then A′BC′ is the required triangle.

To construct the tangents to a circle from a point outside it.

Given : We are given a circle with centre ‘O’ and a point P outside it. We have to construct two tangents from P to the circle.

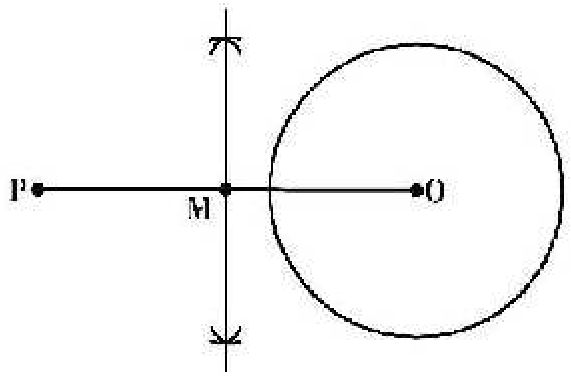
Steps of construction :

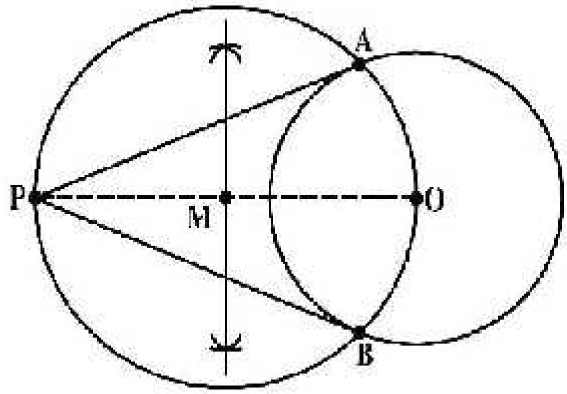
☞ Join PO and draw a perpendicular bisector of it. Let M be the midpoint of PO.

☞ Taking M as centre and PM or MO as radius, draw a circle. Let it intersect the given circle at the points A and B.

☞ Join PA and PB.

Then PA and PB are the required two tangents.





IMPORTANT QUESTIONS FOR PRACTICE

1. Construct an isosceles triangle whose base is 7 cm and altitude 4 cm and then construct another similar triangle whose sides are $\frac{3}{2}$ time the corresponding sides of the isosceles triangle.

2. Construct an isosceles triangle whose base is 8 cm and altitude 4 cm and then another triangle whose sides are $1\frac{1}{2}$ times the corresponding sides of the isosceles triangle.

3. Draw a triangle ABC with side BC = 6 cm, AB = 5 cm and $\angle ABC = 60°$. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle ABC.

4. Draw a triangle ABC with side BC = 7 cm, $\angle B = 45°$, $\angle A = 105°$. Then, construct a triangle whose sides are $\frac{4}{3}$ times the corresponding sides of ΔABC.

5. Draw a right triangle in which the sides (other than hypotenuse) are of lengths 4 cm and 3 cm. Then construct another triangle whose sides are $\frac{5}{3}$ times the corresponding sides of the given triangle.

6. Draw a circle of radius 6 cm. From a point 10 cm away from its centre, construct the pair of tangents to the circle and measure their lengths.

7. Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 cm and measure its length. Also verify the measurement by actual calculation.

8. Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameter each at a distance of 7 cm from its centre. Draw tangents to the circle from these two points P and Q.

9. Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°.

10. Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle.

CHAPTER – 12
AREAS RELATED TO CIRCLES

AREA AND PERIMETER OF CIRCLE, QUADRANT, SEMICIRCLE

Area of Circle = πr^2 , Perimeter of Circle = Circumference = $2\pi r$

Area of Semicircle = $\frac{1}{2}\pi r^2$, Perimeter of Semicircle = $\pi r + 2r$

Area of Quadrant = $\frac{1}{4}\pi r^2$, Perimeter of Quadrant = $\frac{1}{2}\pi r + 2r$

IMPORTANT QUESTONS

Find the diameter of the circle whose area is equal to the sum of the areas of the two circles of diameters 20 cm and 48 cm.

Solution: Here, radius r_1 of first circle = 20/2 cm = 10 cm
and radius r_2 of the second circle = 48/2 cm = 24 cm
Therefore, sum of their areas = $\pi r_1^2 + \pi r_2^2 = \pi (10)^2 + \pi (24)^2 = \pi \times 676$
Let the radius of the new circle be r cm. Its area = πr^2
Therefore, $\pi r^2 = \pi \times 676 \quad \Rightarrow r^2 = 676 \quad \Rightarrow r = 26$
Thus, radius of the new circle = 26 cm
Hence, diameter of the new circle = 2×26 cm = 52 cm

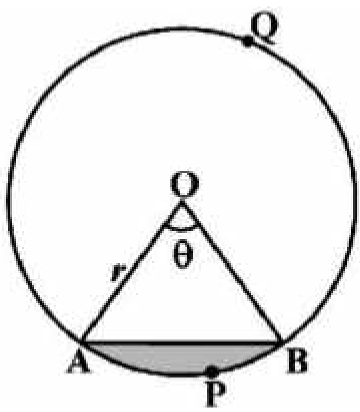
Questions for Practice

1. The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.
2. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.
3. The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?
4. Find the area of a quadrant of a circle whose circumference is 22 cm.

AREAS OF SECTOR AND SEGMENT OF A CIRCLE

Area of the sector of angle θ $= \frac{\theta}{360^0} \times \pi r^2$, where *r* is the radius of the circle and θ the angle of the sector in degrees

length of an arc of a sector of angle θ $= \frac{\theta}{360^0} \times 2\pi r$, where *r* is the radius of the circle and θ the angle of the sector in degrees



Area of the segment APB = Area of the sector OAPB – Area of Δ OAB

$$= \frac{\theta}{360^0} \times \pi r^2 - \text{area of } \Delta \text{ OAB}$$

- ☞ Area of the major sector OAQB = πr^2 – Area of the minor sector OAPB
- ☞ Area of major segment AQB = πr^2 – Area of the minor segment APB
- ☞ Area of segment of a circle = Area of the corresponding sector – Area of the corresponding triangle

IMPORTANT QUESTIONS

Find the area of the sector of a circle with radius 4 cm and of angle 30°. Also, find the area of the corresponding major sector (Use π = 3.14).

Solution : Here, radius, r = 4 cm, ¯ = 30^0,

We know that Area of sector = $\frac{¯}{360^0} \times ® r^2 = \frac{30^0}{360^0} \times 3.14 \times 4 \times 4 = \frac{1}{12} \times 3.14 \times 4 \times 4$

$= \frac{12.56}{3} = 4.19 cm^2$ (approx.)

Area of the corresponding major sector

= πr^2 – area of sector OAPB

= (3.14 × 16 – 4.19) cm^2

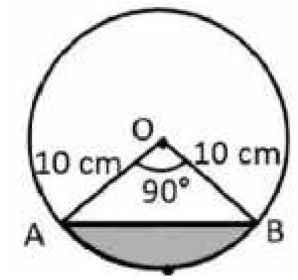
= 46.05 cm2 = 46.1 cm^2(approx.)

A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding : (i) minor segment (ii) major sector. (Use π = 3.14)

Solutions: Here, radius, r = 10 cm, ¯ = 90^0,

We know that Area of minor sector = $\frac{¯}{360^0} \times ® r^2 = \frac{90^0}{360^0} \times 3.14 \times 10 \times 10 = \frac{1}{4} \times 314 = 78.5 cm^2$

and Area of triangle AOB = $\frac{1}{2} \times b \times h = \frac{1}{2} \times 10 \times 10 = 50 cm^2$



Area of minor segment = Area of minor sector –
Area of triangle AOB = 78.5 – 50 = 28.5 cm^2.

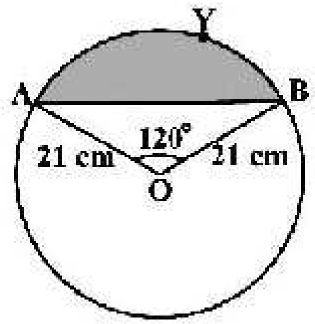
Area of circle = $® r^2 = 3.14 \times 10 \times 10 = 314 cm^2$

Area of major sector = Area of circle – Area of minor sector

= 314 – 78.5 = 235.5 cm^2

Questions for Practice

1. Find the area of the segment AYB shown in below figure, if radius of the circle is 21 cm and ∠AOB = 120°.



2. Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°.

3. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

4. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope. Find (i) the area of that part of the field in which the horse can graze. (ii) the increase in the grazing area if the rope were 10 m long instead of 5 m. (Use π = 3.14)

5. A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors. Find : (i) the total length of the silver wire required. (ii) the area of each sector of the brooch.

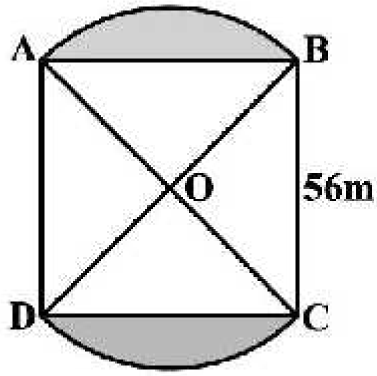
6. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find: (i) the length of the arc (ii) area of the sector formed by the arc (iii) area of the segment formed by the corresponding chord

7. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. (Use π = 3.14 and $\sqrt{3}$ = 1.73)
8. A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use π = 3.14 and $\sqrt{3}$ = 1.73)
9. A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.
10. To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned. (Use π = 3.14)

AREA OF SHADED REGION BASED QUESTIONS

IMPORTANT QUESTIONS

In the adjoining figure, two circular flower beds have been shown on two sides of a square lawn ABCD of side 56 m. If the centre of each circular flower bed is the point of intersection O of the diagonals of the square lawn, find the sum of the areas of the lawn and the flower beds.



Solution: Here, side of square ABCD, a = 56 m

diagonal of square = $a\sqrt{2} = 56\sqrt{2}$

radius, r = OA = OB = OC = OD = $\frac{56\sqrt{2}}{2} = 28\sqrt{2}cm$

Now, Area of sector OAB = Area of sector ODC

$= \frac{-}{360^0} \times ® r^2 = \frac{90^0}{360^0} \times \frac{22}{7} \times r^2 = \frac{1}{4} \times \frac{22}{7} \times r^2$

and Area of Δ OAD = Area of Δ OBC = $\frac{1}{2} \times r \times r = \frac{1}{2} \times r^2$

Total area = Area of sector OAB + Area of sector ODC + Area of Δ OAD + Area of Δ OBC

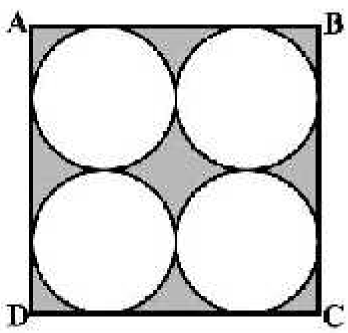
$= \frac{1}{4} \times \frac{22}{7} \times r^2 + \frac{1}{4} \times \frac{22}{7} \times r^2 + \frac{1}{2} \times r^2 + \frac{1}{2} \times r^2$

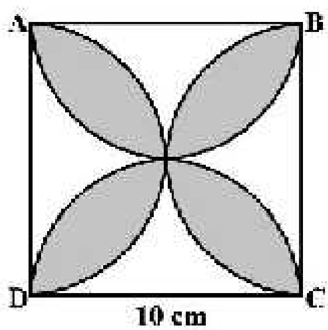
$= 2 \times \frac{1}{4} \times \frac{22}{7} \times r^2 + 2 \times \frac{1}{2} \times r^2 = \frac{11}{7} \times r^2 + r^2 = \left(\frac{11}{7} + 1\right) r^2$

$= \frac{18}{7} \times 28 \times 28 \times 2 = 4032cm^2$

Questions for Practice

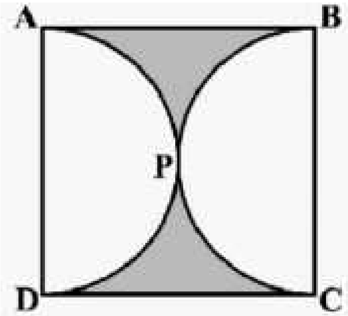
1. Find the area of the shaded region in below left figure, where ABCD is a square of side 14 cm.

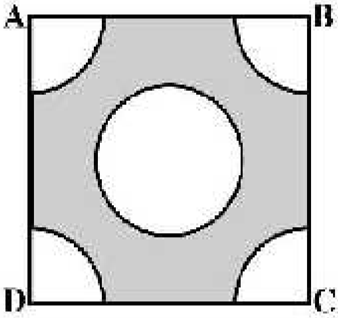




2. Find the area of the shaded design in above right figure, where ABCD is a square of side 10 cm and semicircles are drawn with each side of the square as diameter. (Use π = 3.14)

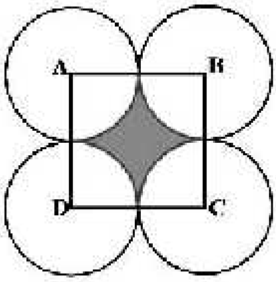
3. Find the area of the shaded region in below left figure, if ABCD is a square of side 14 cm and APD and BPC are semicircles.

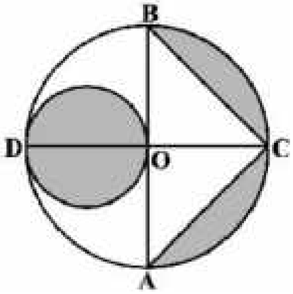




4. From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in above right sided figure. Find the area of the remaining portion of the square.

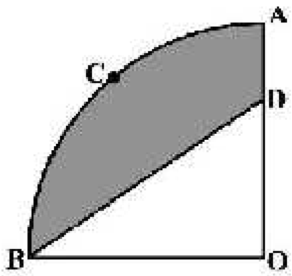
5. In the below left figure, ABCD is a square of side 14 cm. With centres A, B, C and D, four circles are drawn such that each circle touch externally two of the remaining three circles. Find the area of the shaded region.

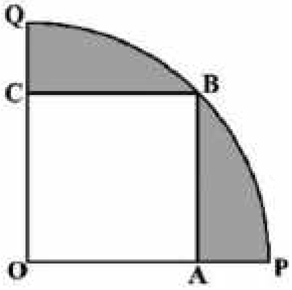




6. In the above right sided figure, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.

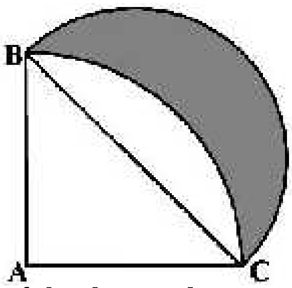
7. In the below left figure, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.

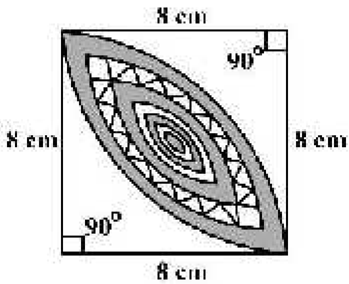




8. In the above right sided figure, OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm, find the area of the (i) quadrant OACB, (ii) shaded region.

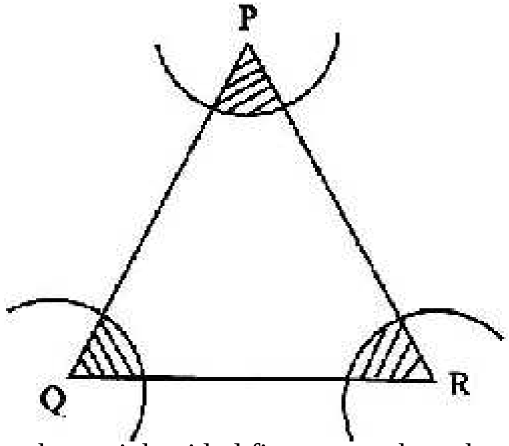
9. In the below figure, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. (Use $\pi = 3.14$)

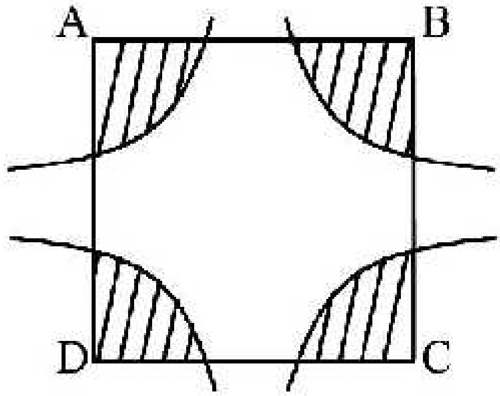




10. Calculate the area of the designed region in above right sided figure, common between the two quadrants of circles of radius 8 cm each.

11. In the below figure, arcs have been drawn with radii 14 cm each and with centres P, Q and R. Find the area of the shaded region.

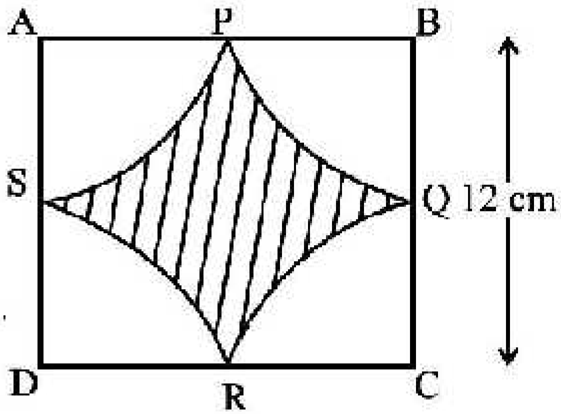


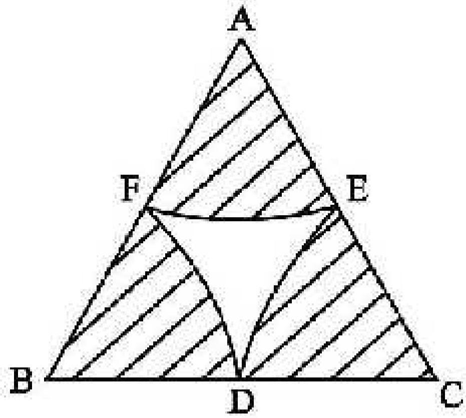


12. In the above right sided figure, arcs have been drawn of radius 21 cm each with vertices A, B, C and D of quadrilateral ABCD as centres. Find the area of the shaded region.

13. A circular park is surrounded by a road 21 m wide. If the radius of the park is 105 m, find the area of the road.

14. Find the area of the shaded region in the below figure, where arcs drawn with centres A, B, C and D intersect in pairs at mid-points P, Q, R and S of the sides AB, BC, CD and DA, respectively of a square ABCD (Use $\pi = 3.14$).





15. In the above right sided figure, arcs are drawn by taking vertices A, B and C of an equilateral triangle of side 10 cm. to intersect the sides BC, CA and AB at their respective mid-points D, E and F. Find the area of the shaded region (Use $\pi = 3.14$).

CHAPTER – 14
STATISTICS

MEAN OF GROUPED DATA

Direct method

Mean, $\bar{x} = \dfrac{\sum f_i x_i}{\sum f_i}$

Assume mean method or Short-cut method

Mean, $\bar{x} = A + \dfrac{\sum f_i d_i}{\sum f_i}$ where $d_i = x_i - A$

Step Deviation method

Mean, $\bar{x} = A + \dfrac{\sum f_i u_i}{\sum f_i} \times h$ where $u = \dfrac{x_i - A}{h}$

IMPORTANT QUESTIONS

The following table gives the literacy rate (in percentage) of 35 cities. Find the mean literacy rate.

Literacy rate (in %)	45 – 55	55 – 65	65 – 75	75 – 85	85 – 95
Number of cities	3	10	11	8	3

Solution:

Literacy rate (in %)	**Number of Cities ‘f’**	**Class mark ‘x’**	$u = \dfrac{x - A}{h}$	**fu**
45 – 55	3	50	–2	–6
55 – 65	10	60	–1	–10
65 – 75	11	70	0	0
75 – 85	8	80	1	8
85 – 95	3	90	2	6
Total	35			–2

Here, $\sum fu = -2, \sum f = 35$, A = 70, h = 10

Mean, $\bar{x} = A + \dfrac{\sum fu}{\sum f} \times h \; = \Rightarrow \bar{x} = 70 + \dfrac{-2}{35} \times 10 = 70 - \dfrac{20}{35} = 70 - \dfrac{4}{7} = 70 - 0.57 \;\Rightarrow \bar{x} = 69.43$

Questions for Practice

1. Find the mean of the following data:

Class Interval	10 – 25	25 – 40	40 – 55	55 – 70	70 – 85	85 – 100
Frequency	2	3	7	6	6	6

2. Find the mean percentage of female teachers of the following data:

Percentage of female teachers	15 – 25	25 – 35	35 – 45	45 – 55	55 – 65	65 – 75	75 – 85
Number of States/U.T	6	11	7	4	4	2	1

3. A survey was conducted by a group of students as a part of their environment awareness programme, in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Number of plants	0 – 2	2 – 4	4 – 6	6 – 8	8 – 10	10 – 12	12 – 14
Number of houses	1	2	1	5	6	2	3

4. Find the mean daily wages of the workers of the factory by using an appropriate method for the following data:

Daily wages (in Rs)	100 – 120	120 – 140	140 – 160	160 – 180	180 – 200
Number of workers	12	14	8	6	10

5. Find the mean number of mangoes kept in a packing box for the following data:

Number of mangoes	50 – 52	53 – 55	56 – 58	59 – 61	62 – 64
Number of boxes	15	110	135	115	25

6. Find the mean daily expenditure on food for the following data:

Daily expenditure (in Rs.)	100 – 150	150 – 200	200 – 250	250 – 300	300 – 350
Number of households	4	5	12	2	2

MODE OF GROUPED DATA

$$Mode = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

where l = lower limit of the modal class,
h = size of the class interval (assuming all class sizes to be equal),
f_1 = frequency of the modal class,
f_0 = frequency of the class preceding the modal class,
f_2 = frequency of the class succeeding the modal class.

IMPORTANT QUESTIONS

Find the mean, mode and median for the following frequency distribution.

Class	0-10	10-20	20-30	30-40	40-50	Total
Frequency	8	16	36	34	6	100

Solution:
Here, highest frequency is 36 which belongs to class 20 – 30. So, modal class is 20 – 30,
l = 20, f_0 = 16, f_1 = 36, f_2 = 34, h = 10

We know that $Mode = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$

$$\Rightarrow Mode = 20 + \frac{36-16}{2(36)-16-34} \times 10$$

$$\Rightarrow Mode = 20 + \frac{20}{72-50} \times 10 = 20 + \frac{200}{22} = 20 + 9.09 = 29.09$$

Questions for Practice

1. The frequency distribution table of agriculture holdings in a village is given below:

Area of land(in ha)	1-3	3-5	5-7	79	9-11	11-13
No. of families	20	45	80	55	40	12

Find the modal agriculture holdings of the village.

2. Find the mode age of the patients from the following distribution :

Age(in years)	6-15	16-25	26-35	36-45	46-55	56-65
No. of patients	6	11	21	23	14	5

3. Find the mode of the following frequency distribution:

Class	25-30	30-35	35-40	40-45	45-50	50-55
Frequency	25	34	50	42	38	14

4. Find the modal height of maximum number of students from the following distribution:

Height(in cm)	160-162	163-165	166-168	169-171	172-174
No. of students	15	118	142	127	18

5. A survey regarding the heights (in cms) of 50 girls of a class was conducted and the following data was obtained.

Height(in cm)	120-130	130-140	140-150	150-160	160-170	**Total**
No. of girls	2	8	12	20	8	**50**

Find the mode of the above data.

- **Cumulative Frequency:** The cumulative frequency of a class is the frequency obtained by adding the frequencies of all the classes preceeding the given class.

MEDIAN OF GROUPED DATA

$$Median = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

where l = lower limit of median class,
n = number of observations,
cf = cumulative frequency of class preceding the median class,
f = frequency of median class,
h = class size (assuming class size to be equal).

EMPIRICAL FORMULA

3Median = Mode + 2 Mean

IMPORTANT QUESTIONS

Find the median of the following frequency distribution:

Class	75-84	85-94	95-104	105-114	115-124	125-134	135-144
Frequency	8	11	26	31	18	4	2

Solution:

Class	**True Class limits**	**Frequency**	**cf**
75-84	74.5 – 84.5	8	8
85-94	84.5 – 94.5	11	19
95-104	94.5 – 104.5	26	45
105-114	104.5 – 114.5	31	76
115-124	114.5 – 124.5	18	94
125-134	124.5 – 134.5	4	98
135-144	134.5 – 144.5	2	100
Total		**100**	

Here, n = 100 $\Rightarrow \frac{n}{2} = 50$ which belongs to 104.5 – 114.5

So, l = 104.5, cf = 45, f = 31, h = 10

We know that $Median = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$

$$\Rightarrow Median = 104.5 + \frac{50-45}{31} \times 10 \quad \Rightarrow Median = 104.5 + \frac{50}{31} = 104.5 + 1.61 = 106.11$$

Questions for Practice

1. The percentage of marks obtained by 100 students in an examination are given below:

Marks	30-35	35-40	40-45	45-50	50-55	55-60	60-65
No. of Students	14	16	18	23	18	8	3

Determine the median percentage of marks.

2. Weekly income of 600 families is as under:

Income(in Rs.)	0-1000	1000-2000	2000-3000	3000-4000	4000-5000	5000-6000
No. of Families	250	190	100	40	15	5

Compute the median income.

3. Find the median of the following frequency distribution:

Marks	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35	35 – 40
Number of students	8	12	20	12	18	13	10	7

4. The following table gives the distribution of the life time of 500 neon lamps:

Life time (in hrs)	1500 – 2000	2000 – 2500	2500 – 3000	3000 – 3500	3500 – 4000	4000 – 4500	4500 – 5000
Number of Lamps	24	86	90	115	95	72	18

Find the median life time of a lamp.

5. Find the median marks for the following distribution:

Marks	Below 10	Below 20	Below 30	Below 40	Below 50	Below 60
No. of Students	6	15	29	41	60	70

6. Find the median wages for the following frequency distribution:

Wages per day	61-70	71-80	81-90	91-100	101-110	111-120
No. of workers	5	15	20	30	10	8

7. Find the median marks for the following distribution:

Marks	11-15	16-20	21-25	26-30	31-35	36-40	41-45	46-50
No. of Students	2	3	6	7	14	12	4	2

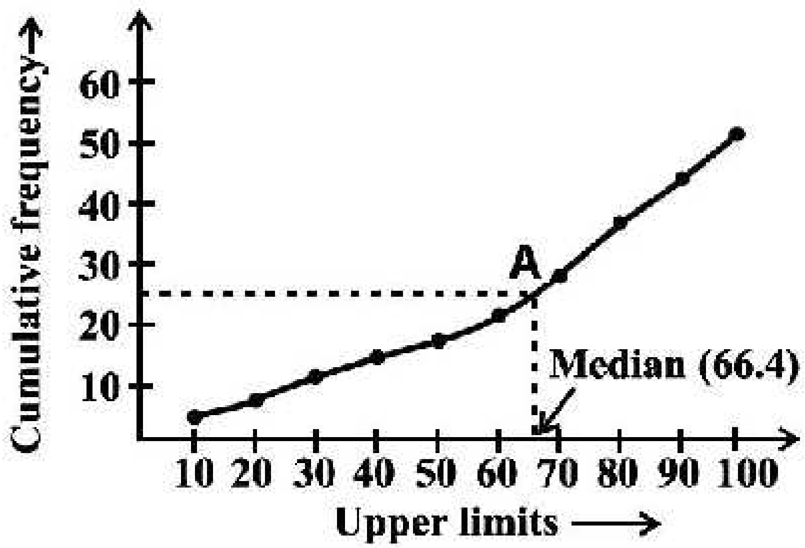
❖ CUMULATIVE FREQUENCY CURVE IS ALSO KNOWN AS 'OGIVE'.

There are three methods of drawing ogive:

1. LESS THAN METHOD

Steps involved in calculating median using less than Ogive approach-

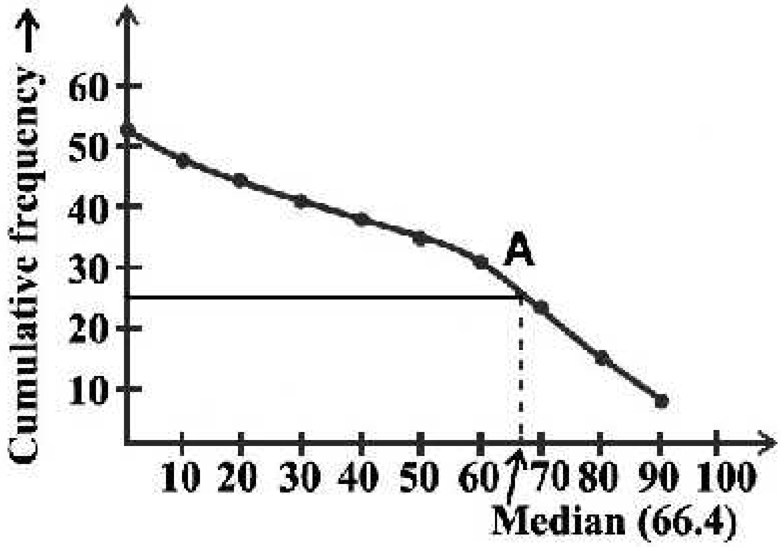
- Convert the series into a 'less than ' cumulative frequency distribution.
- Let N be the total number of students who's data is given. N will also be the cumulative frequency of the last interval. Find the $(N/2)^{th}$ itemand mark it on the y-axis.
- Draw a perpendicular from that point to the right to cut the Ogive curve at point A.
- From point A where the Ogive curve is cut, draw a perpendicular on the x-axis. The point at which it touches the x-axis will be the median value of the series as shown in the graph.



2. MORE THAN METHOD

Steps involved in calculating median using more than Ogive approach-

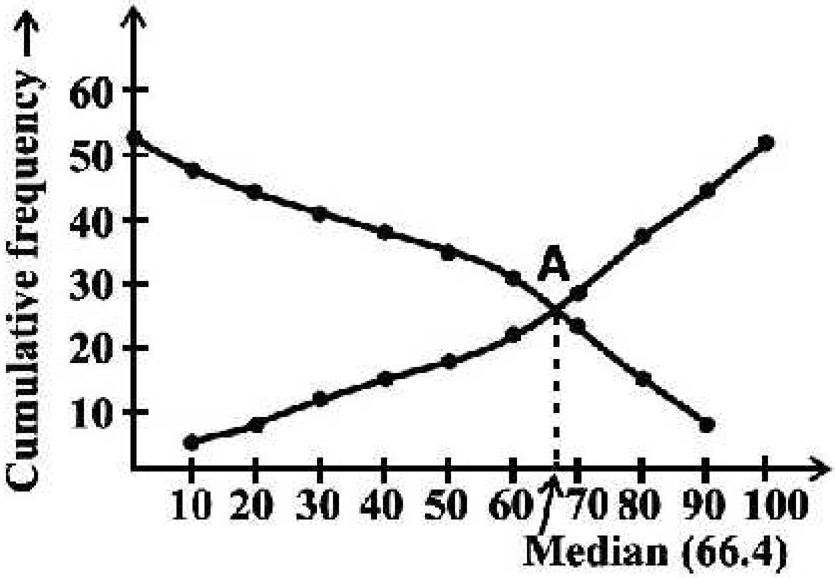
- Convert the series into a 'more than ' cumulative frequency distribution.
- Let N be the total number of students who's data is given. N will also be the cumulative frequency of the last interval. Find the $(N/2)^{th}$ item and mark it on the y-axis.
- Draw a perpendicular from that point to the right to cut the Ogive curve at point A.
- From point A where the Ogive curve is cut, draw a perpendicular on the x-axis. The point at which it touches the x-axis will be the median value of the series as shown in the graph.



3. LESS THAN AND MORE THAN OGIVE METHOD

Another way of graphical determination of median is through simultaneous graphic presentation of both the less than and more than Ogives.

- Mark the point A where the Ogive curves cut each other.
- Draw a perpendicular from A on the x-axis. The corresponding value on the x-axis would be the median value.



❖ The median of grouped data can be obtained graphically as the *x*-coordinate of the point of intersection of the two ogives for this data.

IMPORTANT QUESTIONS

The following distribution gives the daily income of 50 workers of a factory.

Daily income (in Rs)	100 – 120	120 – 140	140 –160	160 –180	180 –200
Number of workers	12	14	8	6	10

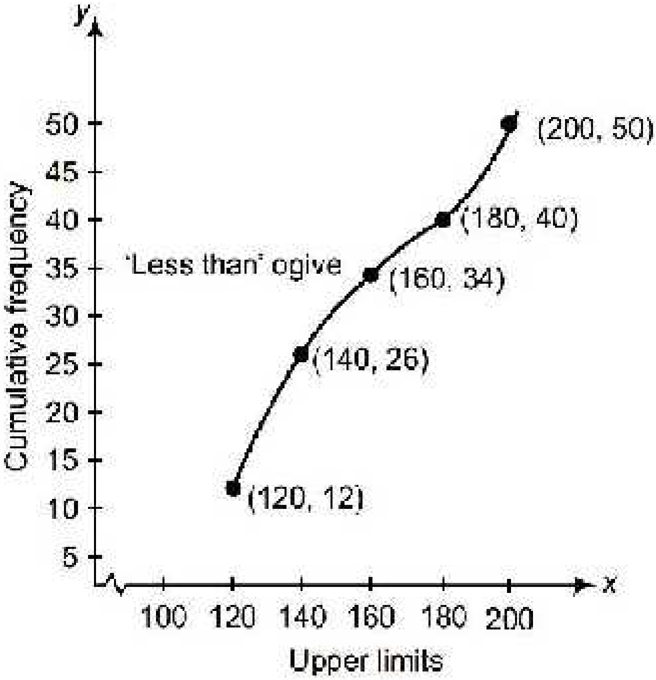
Convert the distribution above to a less than type cumulative frequency distribution, and draw its ogive.

Solution:

Cumulative frequency less than type

Daily income (in Rs)	**Less than type cf**
Less than 120	12
Less than 140	26
Less than 160	34
Less than 180	40
Less than 200	50

On the graph, we will plot the points (120, 12), (140, 26), (160, 34), (180, 40) and (200, 50).



Questions for Practice

1. The following table gives production yield per hectare of wheat of 100 farms of a village.

Production yield (in kg/ha)	50 - 55	55 - 60	60 - 65	65 - 70	70 - 75	75 - 80
Number of farms	2	8	12	24	38	16

Change the distribution to a more than type distribution, and draw its ogive.

2. For the following distribution, draw the cumulative frequency curve more than type and hence obtain the median from the graph.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	5	15	20	23	17	11	9

3. Draw less than ogive for the following frequency distribution:

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Number of students	5	8	6	10	6	6

Also find the median from the graph and verify that by using the formula.

4. The table given below shows the frequency distribution of the cores obtained by 200 candidates in a BCA examination.

Score	200-250	250-300	300-350	350-400	400-450	450-500	500-550	550-600
No. of students	30	15	45	20	25	40	10	15

Draw cumulative frequency curves by using (i) less than type and (ii) more than type. Hence find median

5. Draw less than and more than ogive for the following frequency distribution:

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Number of students	8	5	10	6	6	6

Also find the median from the graph and verify that by using the formula.

CHAPTER – 15
PROBABILITY

PROBABILITY

The theoretical probability (also called classical probability) of an event A, written as P(A), is defined as

$$P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Number of all possible outcomes of the experiment}}$$

COMPLIMENTARY EVENTS AND PROBABILITY

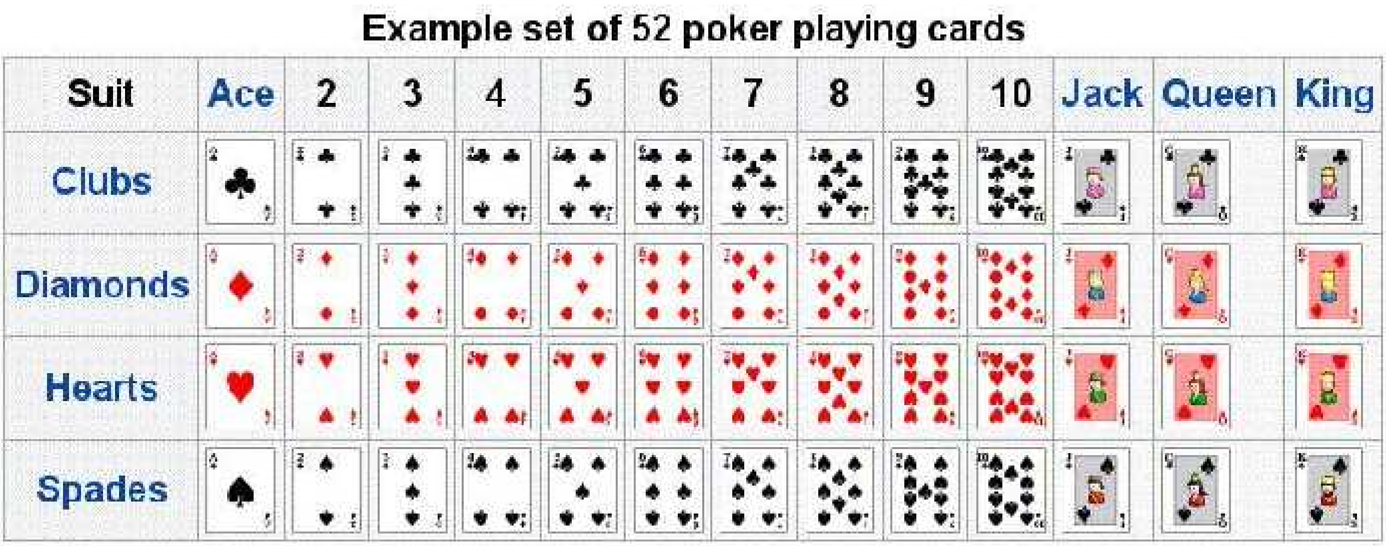
We denote the event 'not E' by E . This is called the **complement** event of event E.
So, P(E) + P(not E) = 1
i.e., $P(E) + P(\overline{E}) = 1$, which gives us $P(\overline{E}) = 1 - P(E)$.

- ☞ The probability of an event which is impossible to occur is 0. Such an event is called an **impossible event**.
- ☞ The probability of an event which is sure (or certain) to occur is 1. Such an event is called a **sure event** or a **certain event**.
- ☞ The probability of an event E is a number P(E) such that $0 \le P(E) \le 1$
- ☞ An event having only one outcome is called an elementary event. The sum of the probabilities of all the elementary events of an experiment is 1.

DECK OF CARDS AND PROBABILITY

A deck of playing cards consists of 52 cards which are divided into 4 suits of 13 cards each. They are black spades (♠) red hearts (♥), red diamonds (♦) and black clubs (♣).

The cards in each suit are Ace, King, Queen, Jack, 10, 9, 8, 7, 6, 5, 4, 3 and 2. Kings, Queens and Jacks are called face cards.



IMPORTANT QUESTIONS

Two dice are thrown together. Find the probability that the sum of the numbers on the top of the dice is (i) 9 (ii) 10

Solution:

Here, total number of outcomes, n(s) = 36

(i) Let A be the event of getting the sum of the numbers on the top of the dice is 9 then we have n(A) = 4 i.e. (3, 6), (4, 5), (5, 4), (6, 3)

Therefore, Probability of getting the sum of the numbers on the top of the dice is 9, $P(A) = \frac{n(A)}{n(S)}$

$\Rightarrow P(A) = \frac{4}{36} = \frac{1}{9}$

(ii) Let B be the event of getting the sum of the numbers on the top of the dice is 10 then we have n(B) = 3 i.e. (4, 6), (5, 5), (6, 4)

Therefore, Probability of getting the sum of the numbers on the top of the dice is 10, $P(B) = \frac{n(B)}{n(S)}$

$\Rightarrow P(B) = \frac{3}{36} = \frac{1}{12}$

One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting (i) red colour ace card (ii) a face card or a spade card (iii) a black face card

Solution:

Here, total number of outcomes, n(s) = 52

(i) Let A be the event of getting red colour ace card and we know that the number of red ace card is 2 then we have, n(A) = 2

Therefore, Probability of getting red colour ace card, $P(A) = \frac{n(A)}{n(S)}$

$\Rightarrow P(A) = \frac{2}{52} = \frac{1}{26}$

(ii) Let B be the event of getting a face card or a spade card and we know that there are 12 face cards, 13 spade cards and 3 face cards are spade then we have, n(B) = 12 + 13 – 3 = 22

Therefore, Probability of getting a face card or a spade card, $P(B) = \frac{n(B)}{n(S)}$

$\Rightarrow P(B) = \frac{22}{52} = \frac{11}{26}$

(ii) Let B be the event of getting a black face card and we know that there are 6 face cards are black then we have, n(C) = 6

Therefore, Probability of getting a black face card, $P(C) = \frac{n(C)}{n(S)}$

$\Rightarrow P(C) = \frac{6}{52} = \frac{3}{26}$

Questions for Practice

1. Two dice are thrown together. Find the probability that the product of the numbers on the top of the dice is (i) 6 (ii) 12 (iii) 7
2. A die is thrown twice. What is the probability that (i) 5 will not come up either time? (ii) 5 will come up at least once?
3. A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that (i) She will buy it ? (ii) She will not buy it ?

4. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting (i) a king of red colour (ii) a face card (iii) a red face card (iv) the jack of hearts (v) a spade (vi) the queen of diamonds

5. Five cards—the ten, jack, queen, king and ace of diamonds, are well-shuffled with their face downwards. One card is then picked up at random. (i) What is the probability that the card is the queen? (ii) If the queen is drawn and put aside, what is the probability that the second card picked up is (a) an ace? (b) a queen?

6. 12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen taken out is a good one.

7. A piggy bank contains hundred 50p coins, fifty Re 1 coins, twenty Rs 2 coins and ten Rs 5 coins. If it is equally likely that one of the coins will fall out when the bank is turned upside down, what is the probability that the coin (i) will be a 50 p coin ? (ii) will not be a Rs 5 coin?

8. A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be (i) red ? (ii) white ? (iii) not green?

9. (i) A lot of 20 bulbs contain 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective?

(ii) Suppose the bulb drawn in (i) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective ?

10. A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears (i) a two-digit number (ii) a perfect square number (iii) a number divisible by 5.

11. A carton consists of 100 shirts of which 88 are good, 8 have minor defects and 4 have major defects. Jimmy, a trader, will only accept the shirts which are good, but Sujatha, another trader, will only reject the shirts which have major defects. One shirt is drawn at random from the carton. What is the probability that (i) it is acceptable to Jimmy? (ii) it is acceptable to Sujatha?

12. Two customers are visiting a particular shop in the same week (Monday to Saturday). Each is equally likely to visit the shop on any day as on another day. What is the probability that both will visit the shop on (i) the same day? (ii) consecutive days? (iii) different days?

13. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is *double* that of a red ball, determine the number of blue balls in the bag.

14. A box contains 12 balls out of which x are black. If one ball is drawn at random from the box, what is the probability that it will be a black ball? If 6 more black balls are put in the box, the probability of drawing a black ball is now double of what it was before. Find x.

15. A jar contains 24 marbles, some are green and others are blue. If a marble is drawn at random from the jar, the probability that it is green is $\frac{2}{3}$. Find the number of blue marbles in the jar.

CHAPTER – 3
PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

ALGEBRAIC INTERPRETATION OF PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

The pair of linear equations represented by these lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

1. If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ then the pair of linear equations has exactly one solution.

2. If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ then the pair of linear equations has infinitely many solutions.

3. If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ then the pair of linear equations has no solution.

S. No.	**Pair of lines**	**Compare the ratios**	**Graphical representation**	**Algebraic interpretation**
1	$a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Unique solution (Exactly one solution)
2	$a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many solutions
3	$a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution

IMPORTANT QUESTIONS

1. On comparing the ratios $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, find out whether the lines representing the following pairs of linear equations intersect at a point, are parallel or coincident:
(i) $5x – 4y + 8 = 0$ and $7x + 6y – 9 = 0$ (ii) $9x + 3y + 12 = 0$ and $18x + 6y + 24 = 0$
(iii) $6x – 3y + 10 = 0$ and $2x – y + 9 = 0$.

2. On comparing the ratios $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent, or inconsistent.
(i) $3x + 2y = 5$; $2x – 3y = 7$ (ii) $2x – 3y = 8$; $4x – 6y = 9$
(iii) $5x – 3y = 11$; $– 10x + 6y = –22$

3. Find the number of solutions of the following pair of linear equations:
$x + 2y – 8 = 0$
$2x + 4y = 16$

4. Write whether the following pair of linear equations is consistent or not.
$x + y = 14$, $x – y = 4$

5. Given the linear equation $3x + 4y – 8 = 0$, write another linear equation in two variables such that the geometrical representation of the pair so formed is parallel lines.

6. Find the value of k so that the following system of equations has no solution:
$3x – y – 5 = 0$, $6x – 2y + k = 0$

7. Find the value of k so that the following system of equation has infinite solutions:
$3x – y – 5 = 0$, $6x – 2y + k = 0$

8. For which values of p, does the pair of equations given below has unique solution?
$4x + py + 8 = 0$ and $2x + 2y + 2 = 0$

9. Determine k for which the system of equations has infinite solutions:
$4x + y = 3$ and $8x + 2y = 5k$

10. Find whether the lines representing the following pair of linear equations intersect at a point, are parallel or coincident:
$2x - 3y + 6 = 0; 4x - 5y + 2 = 0$

11. Find the value of k for which the system $3x + ky = 7$, $2x - 5y = 1$ will have infinitely many solutions.

12. For what value of k, the system of equations $2x - ky + 3 = 0$, $4x + 6y - 5 = 0$ is consistent?

13. For what value of k, the system of equations $kx - 3y + 6 = 0$, $4x - 6y + 15 = 0$ represents parallel lines?

14. For what value of p, the pair of linear equations $5x + 7y = 10$, $2x + 3y = p$ has a unique solution.

15. Find the value of m for which the pair of linear equations has infinitely many solutions.
$2x + 3y - 7 = 0$ and $(m - 1)x + (m + 1)y = (3m - 1)$

16. For what value of p will the following pair of linear equations have infinitely many solutions?
$(p - 3)x + 3y = p$; $px + py = 12$

17. For what value of k will the system of linear equations has infinite number of solutions?
$kx + 4y = k - 4$, $16x + ky = k$

18. Find the values of a and b for which the following system of linear equations has infinite number of solutions:
$2x - 3y = 7$, $(a + b)x - (a + b - 3)y = 4a + b$

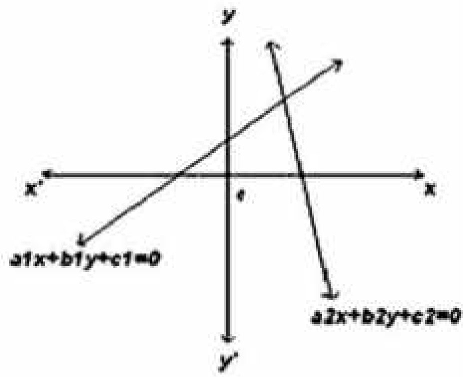
19. For what value of k will the equations $x + 2y + 7 = 0$, $2x + ky + 14 = 0$ represent coincident lines?

20. For what value of k, the following system of equations $2x + ky = 1$, $3x - 5y = 7$ has (i) a unique solution (ii) no solution

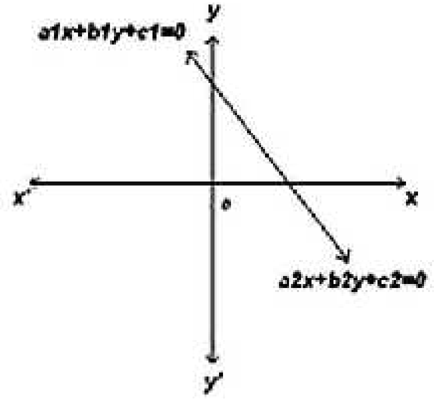
GRAPHICAL METHOD OF SOLUTION OF A PAIR OF LINEAR EQUATIONS

The graph of a pair of linear equations in two variables is represented by two lines.

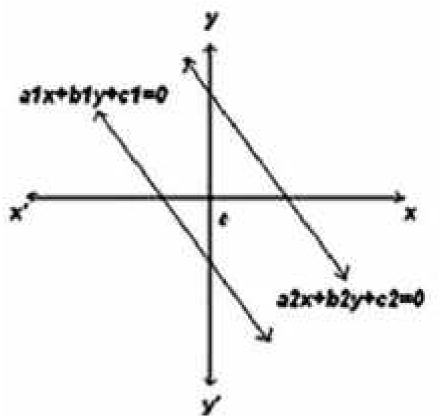
1. If the lines intersect at a point, then that point gives the unique solution of the two equations. In this case, the pair of equations is **consistent**.



2. If the lines coincide, then there are infinitely many solutions — each point on the line being a solution. In this case, the pair of equations is **dependent (consistent)**.



3. If the lines are parallel, then the pair of equations has no solution. In this case, the pair of equations is **inconsistent**.



IMPORTANT QUESTIONS

Solve the equation graphically: x + 3y = 6 and 2x – 3y = 12.

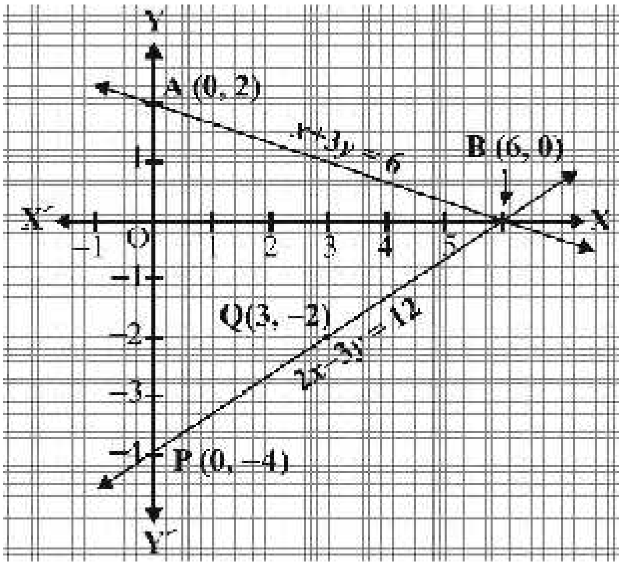
Solution: Given that

$x+3y=6 \Rightarrow 3y=6-x \Rightarrow y=\dfrac{6-x}{3}$

x	0	3	6
y	2	1	0

and $2x-3y=12 \Rightarrow 3y=2x-12 \Rightarrow y=\dfrac{2x-12}{3}$

x	0	3	6
y	-4	-2	0



Now plot the points and join the points to form the lines AB and PQ as shown in graph

Since point B(6, 0) common to both the lines AB and PQ. Therefore, the solution of the pair of linear equations is x = 6 and y = 0

Questions for Practice

1. Determine by drawing graphs, whether the following pair of linear equations has a unique solution or not: 3x + 4y = 12; y = 2
2. Determine by drawing graphs, whether the following pair of linear equations has a unique solution or not: 2x – 5 = 0, y + 4 = 0.
3. Draw the graphs of the equations 4x – y – 8 = 0 and 2x – 3y + 6 = 0. Also, determine the vertices of the triangle formed by the lines and x-axis.
4. Solve the following system of linear equations graphically:3x – 2y – 1 = 0; 2x – 3y + 6 = 0. Shade the region bounded by the lines and x-axis.
5. Solve graphically: x + 4y = 10, y – 2 = 0
6. Solve graphically: 2x – 3y = 6, x – 6 = 0
7. Solve the following system of equations graphically: 3x – 5y + 1 = 0, 2x – y + 3 = 0. Also find the points where the lines represented by the given equations intersect the x-axis.
8. Solve the following system of equations graphically: x – 5y = 6, 2x – 10y = 10 Also find the points where the lines represented by the given equations intersect the x-axis.
9. Solve the following pair of linear equations graphically: x + 3y = 6; 2x – 3y = 12 Also find the area of the triangle formed by the lines representing the given equations with y-axis.

CHAPTER – 4
QUADRATIC EQUATIONS

FACTORISATION METHODS TO FIND THE SOLUTION OF QUADRATIC EQUATIONS

Steps to find the solution of given quadratic equation by factorisation

- Firstly, write the given quadratic equation in standard form $ax^2 + bx + c = 0$.
- Find two numbers ~ and such that sum of ~ and is equal to b and product of ~ and is equal to ac.
- Write the middle term bx as $\sim x + \ x$ and factorise it by splitting the middle term and let factors are (x + p) and (x + q) i.e. $ax^2 + bx + c = 0 \Rightarrow (x + p)(x + q) = 0$
- Now equate reach factor to zero and find the values of x.
- These values of x are the required roots/solutions of the given quadratic equation.

IMPORTANT QUESTIONS

Solve the quadratic equation by using factorization method: $x^2 + 2x - 8 = 0$

Solution: $x^2 + 2x - 8 = 0$

$\Rightarrow x^2 + 4x - 2x - 8 = 0 \Rightarrow x(x + 4) - 2(x + 4) = 0$

$\Rightarrow (x + 4)(x - 2) = 0 \Rightarrow x + 4 = 0, x - 2 = 0 \Rightarrow x = -4, 2$

Questions for practice

1. Solve the quadratic equation using factorization method: $x^2 + 7x - 18 = 0$
2. Solve the quadratic equation using factorization method: $x^2 + 5x - 6 = 0$
3. Solve the quadratic equation using factorization method: $y^2 - 4y + 3 = 0$
4. Solve the quadratic equation using factorization method: $x^2 - 21x + 108 = 0$
5. Solve the quadratic equation using factorization method: $x^2 - 11x - 80 = 0$
6. Solve the quadratic equation using factorization method: $x^2 - x - 156 = 0$
7. Solve the following for x : $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$.
8. Solve the following for x : $\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$

NATURE OF ROOTS

The roots of the quadratic equation $ax^2 + bx + c = 0$ by quadratic formula are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{D}}{2a}$$

where D = $b^2 - 4ac$ is called discriminant. The nature of roots depends upon the value of discriminant D. There are three cases –

Case – I

When D > 0 i.e. $b^2 - 4ac > 0$, then the quadratic equation has two distinct roots.

i.e. $x = \frac{-b + \sqrt{D}}{2a}$ *and* $\frac{-b - \sqrt{D}}{2a}$

Case – II

When D = 0, then the quadratic equation has two equal real roots.

i.e. $x = \frac{-b}{2a}$ *and* $\frac{-b}{2a}$

Case – III

When D < 0 then there is no real roots exist.

IMPORTANT QUESTIONS

Find the discriminant of the quadratic equation 2x2 – 4x + 3 = 0, and hence find the nature of its roots.

Solution : The given equation is of the form $ax^2 + bx + c = 0$, where a = 2, b = – 4 and c = 3.
Therefore, the discriminant, $D = b^2 - 4ac = (-4)^2 - (4 \times 2 \times 3) = 16 - 24 = -8 < 0$
So, the given equation has no real roots.

Questions for Practice

1. Find the discriminant and the nature of the roots of quadratic equation: $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0$.
2. Find discriminant and the nature of the roots of quadratic equation: $4x^2 - 2x2 + 3 = 0$.
3. Find discriminant and the nature of the roots of quadratic equation: $4x^2 - 12x + 9 = 0$.
4. Find discriminant and the nature of the roots of quadratic equation: $5x^2 + 5x + 6 = 0$.
5. Write the nature of roots of quadratic equation $4x^2 + 4\sqrt{3}x + 3 = 0$.
6. Write the nature of roots of the quadratic equation $9x^2 - 6x - 2 = 0$.
7. Write the nature of roots of quadratic equation : $4x^2 + 6x + 3 = 0$
8. The roots of $ax^2 + bx + c = 0$, $a \neq 0$ are real and unequal. What is value of D?
9. If $ax^2 + bx + c = 0$ has equal roots, what is the value of c?

QUADRATIC FORMULA METHOD

Steps to find the solution of given quadratic equation by quadratic formula method:

- Firstly, write the given quadratic equation in standard form $ax^2 + bx + c = 0$.
- Write the values of a, b and c by comparing the given equation with standard form.
- Find discriminant $D = b^2 - 4ac$. If value of D is negative, then is no real solution i.e. solution does not exist. If value of $D \geq 0$, then solution exists follow the next step.
- Put the value of a, b and D in quadratic formula $x = \frac{-b \pm \sqrt{D}}{2a}$ and get the required roots/solutions.

IMPORTANT QUESTIONS

Solve the quadratic equation by using quadratic formula: $x^2 + x - 6 = 0$

Solution: Here, a = 1, b = 1, c = –6

$\Rightarrow D = b^2 - 4ac = 1 - 4(1)(-6) = 1 + 24 = 25 > 0$

Now, $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{25}}{2(1)} = \frac{-1 \pm 5}{2} \Rightarrow x = \frac{-1-5}{2} or \frac{-1+5}{2} \Rightarrow x = \frac{-6}{2} or \frac{4}{2} \Rightarrow x = -3 \ or \ 2$

Questions for practice

1. Solve the quadratic equation by using quadratic formula: $x^2 - 7x + 18 = 0$
2. Solve the quadratic equation by using quadratic formula: $x^2 - 5x + 6 = 0$
3. Solve the quadratic equation by using quadratic formula: $y^2 + 4y + 3 = 0$
4. Solve the quadratic equation by using quadratic formula: $x^2 + 11x - 80 = 0$
5. Solve the quadratic equation by using quadratic formula: $x^2 + x - 156 = 0$
6. Solve for x by using quadratic formula : $9x^2 - 9(a + b)x + (2a^2 + 5ab + 2b^2) = 0$.

WORD PROBLEMS

IMPORTANT QUESTIONS

A motor boat whose speed is 18 km/h in still water takes 1 hr. more to go 24 km upstream than to return downstream to the same spot. Find the speed of stream.

Solution: Let the speed of the stream be x km/h.
Therefore, the speed of the boat upstream = (18 – x) km/h and the speed of the boat downstream = (18 + x) km/h.

The time taken to go upstream = $\frac{\text{distance}}{\text{speed}} = \frac{24}{18-x}$

Similarly, the time taken to go downstream = $\frac{24}{18+x}$

According to the question, $\frac{24}{18-x} - \frac{24}{18+x} = 1$

$\Rightarrow$ 24(18 + x) – 24(18 – x) = (18 – x) (18 + x)

$\Rightarrow$ $x^2 + 48x - 324 = 0$

$\Rightarrow$ (x – 6)(x + 54) = 0 (using factorisation)

$\Rightarrow$ x = 6, –54

Since x is the speed of the stream, it cannot be negative. So, we ignore the root x = – 54. Therefore, x = 6 gives the speed of the stream as 6 km/h.

Questions for Practice

1. In a class test, the sum of the marks obtained by P in Mathematics and Science is 28. Had he got 3 more marks in maths and 4 marks less in science, the product of marks obtained in the two subjects would have been 180. Find the marks obtained in the two subjects separately.

2. A peacock is sitting on the top of a pillar which is 9 m high. From a point 27 m away from the bottom of the pillar, a snake is coming to its hole at the base of the pillar. Seeing the snake the peacock pounces on it. If their speeds are equal at what distance from the hole is the snake caught?

3. Some students planned a picnic. The total budget for food was Rs. 2,000. But 5 students failed to attend the picnic and thus the cost of food for each member increased by Rs. 20. How many students attended the picnic and how much did each student pay for the food?

4. In a flight of 2800 km, an aircraft was slowed down due to bad weather. Its average speed is reduced by 100 km/h and time increased by 30 minutes. Find the original duration of the flight.

5. A takes 6 days less than the time taken by B to finish a piece of work. If both A and B together can finish it in 4 days, find the time taken by B to finish the work.

6. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

7. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

8. Two water taps together can fill a tank is 6 hours. The tap of larger diameter takes 9 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

9. A takes 10 days less than the time taken by B to finish a piece of work. If both A and B together can finish the work in 12 days, find the time taken by B to finish the work.

10. A man bought a certain number of toys for 180, he kept one for his own use and sold the rest for one rupee each more than he gave for them, besides getting his own toy for nothing he made a profit of 10. Find the number of toys.

11. Nine times the side of one square exceeds a perimeter of a second square by one metre and six times the area of the second square exceeds twenty nine times the area of the first by one square metre. Find the side of each square.

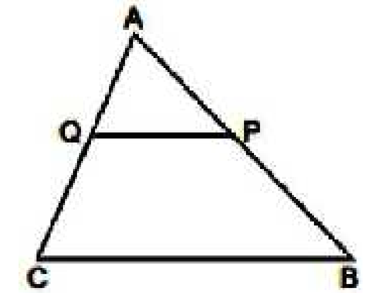
12. One-fourth of a herd of camels was seen in a forest. Twice the square root of the herd had gone to mountains and the remaining 15 camels were seen on the bank of a river. Find the total number of camels.

13. One pipe can fill a cistern in (x + 2) hours and the other pipe can fill the same cistern in (x + 7) hours. If both the pipes, when opened together take 6 hours to fill the empty cistern, find the value of x.

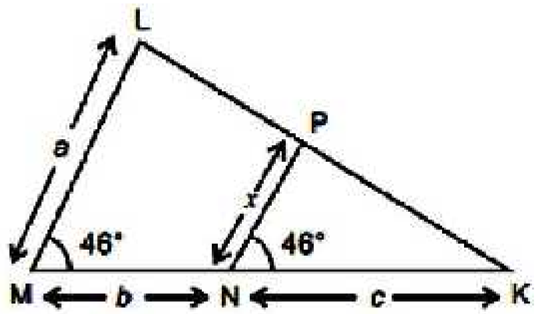
CHAPTER – 6
TRIANGLES

IMPORTANT 1 MARK QUESTIONS

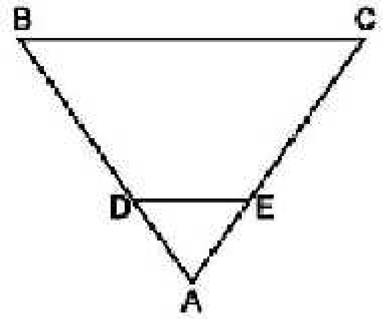
1. In ΔABC, D and E are points on sides AB and AC respectively such that DE || BC and AD : DB = 3 : 1. If EA = 6.6 cm then find AC.
2. In the fig., P and Q are points on the sides AB and AC respectively of ΔABC such that AP = 3.5 cm, PB = 7 cm, AQ = 3 cm and QC = 6 cm. I f PQ = 4.5 cm, find BC.



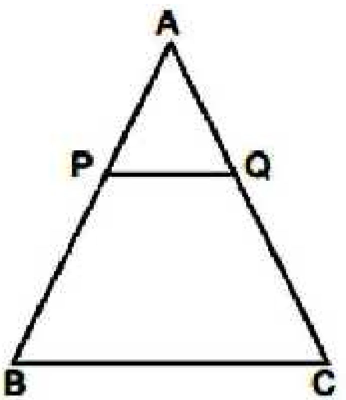
3. The perimeter of two similar triangles ABC and LMN are 60 cm and 48 cm respectively. If LM = 8 cm, then what is the length of AB ?
4. In fig. $\angle M = \angle N = 46°$, express x in terms of a, b and c, where a, b and c are lengths of LM, MN and NK respectively.



5. In figure, DE || BC in ΔABC such that BC = 8 cm, AB = 6 cm and DA = 1.5 cm. Find DE.



6. In the fig., PQ || BC and AP : PB = 1 : 2. Find $\dfrac{ar(\Delta APQ)}{ar(\Delta ABC)}$

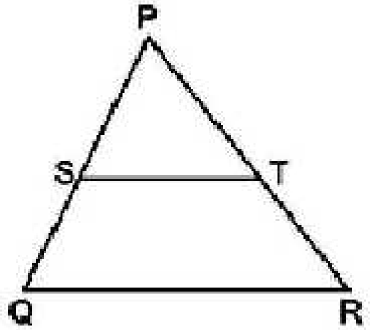


7. A vertical stick 12 m long casts a shadow 8 m long on the ground. At the same time a tower casts the shadow 40 m long on the ground. Determine the height of the tower.

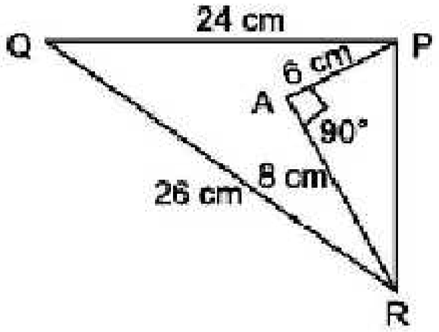
8. If ΔABC and ΔDEF are similar triangles such that $\angle A = 57°$ and $\angle E = 83°$. Find C.

9. If the areas of two similar triangles are in ratio 25 : 64, write the ratio of their corresponding sides.

10. In figure, S and T are points on the sides PQ and PR, respectively of ΔPQR, such that PT = 2 cm, TR = 4 cm and ST is parallel to QR. Find the ratio of the areas of ΔPST and ΔPQR.

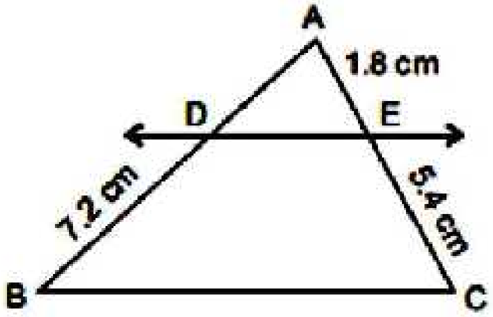


11. In the fig., PQ = 24 cm, QR = 26 cm, $\angle PAR = 90°$, PA = 6 cm and AR = 8 cm. Find $\angle QPR$.



12. The lengths of the diagonals of a rhombus are 30 cm and 40 cm. Find the side of the rhombus.

13. In the given figure, DE ∥ BC. Find AD.



14. The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of first triangle is 9 cm., what is the corresponding side of the other triangle ?

CHAPTER – 7
COORDINATE GEOMETRY

DISTANCE FORMULA

The distance between any two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$or \quad AB = \sqrt{(\text{difference of abscissae})^2 + (\text{difference of ordinates})^2}$$

Distance of a point from origin

The distance of a point P(x, y) from origin O is given by OP = $\sqrt{x^2 + y^2}$

Problems based on geometrical figure

To show that a given figure is a

- ☞ Parallelogram – prove that the opposite sides are equal
- ☞ Rectangle – prove that the opposite sides are equal and the diagonals are equal.
- ☞ Parallelogram but not rectangle – prove that the opposite sides are equal and the diagonals are not equal.
- ☞ Rhombus – prove that the four sides are equal
- ☞ Square – prove that the four sides are equal and the diagonals are equal.
- ☞ Rhombus but not square – prove that the four sides are equal and the diagonals are not equal.
- ☞ Isosceles triangle – prove any two sides are equal.
- ☞ Equilateral triangle – prove that all three sides are equal.
- ☞ Right triangle – prove that sides of triangle satisfies Pythagoras theorem.

IMPORTANT QUESTIONS

Show that the points (1, 7), (4, 2), (–1, –1) and (– 4, 4) are the vertices of a square.

Solution : Let A(1, 7), B(4, 2), C(–1, –1) and D(– 4, 4) be the given points.

$$AB = \sqrt{(1-4)^2 + (7-2)^2} = \sqrt{9+25} = \sqrt{34}$$

$$BC = \sqrt{(4+1)^2 + (2+1)^2} = \sqrt{25+9} = \sqrt{34}$$

$$CD = \sqrt{(-1+4)^2 + (-1-4)^2} = \sqrt{9+25} = \sqrt{34}$$

$$DA = \sqrt{(1+4)^2 + (7-4)^2} = \sqrt{25+9} = \sqrt{34}$$

$$AC = \sqrt{(1+1)^2 + (7+1)^2} = \sqrt{4+64} = \sqrt{68}$$

$$BD = \sqrt{(4+4)^2 + (2-4)^2} = \sqrt{64+4} = \sqrt{68}$$

Since, AB = BC = CD = DA and AC = BD, all the four sides of the quadrilateral ABCD are equal and its diagonals AC and BD are also equal. Therefore, ABCD is a square.

Find a point on the y-axis which is equidistant from the points A(6, 5) and B(– 4, 3).

Solution : We know that a point on the y-axis is of the form (0, y). So, let the point P(0, y) be equidistant from A and B. Then $AP^2 = BP^2$

$\Rightarrow (6 - 0)^2 + (5 - y)^2 = (-4 - 0)^2 + (3 - y)^2$

$\Rightarrow 36 + 25 + y^2 - 10y = 16 + 9 + y^2 - 6y \Rightarrow 4y = 36 \Rightarrow y = 9$

So, the required point is (0, 9).

Questions for practice

1. Show that the points A(1, 2), B(5, 4), C(3, 8) and D(–1, 6) are vertices of a square.
2. Show that the points A(5, 6), B(1, 5), C(2, 1) and D(6, 2) are vertices of a square.
3. Show that the points A(1, –3), B(13, 9), C(10, 12) and D(–2, 0) are vertices of a rectangle.
4. Show that the points A(1, 0), B(5, 3), C(2, 7) and D(–2, 4) are vertices of a rhombus.
5. Prove that the points A(–2, –1), B(1, 0), C(4, 3) and D(1, 2) are vertices of a parallelogram.
6. Find the point on x-axis which is equidistant from (7, 6) and (–3, 4).
7. Find the point on the x-axis which is equidistant from (2, –5) and (–2, 9).
8. Find a point on the y-axis which is equidistant from the points A(5, 2) and B(– 4, 3).
9. Find a point on the y-axis which is equidistant from the points A(5, – 2) and B(– 3, 2).
10. Find the values of y for which the distance between the points P(2, – 3) and Q(10, y) is 10 units.
11. Find the value of a , if the distance between the points A (–3, –14) and B (a, –5) is 9 units.
12. If the point A (2, – 4) is equidistant from P (3, 8) and Q (–10, y), find the values of y. Also find distance PQ.

<u>Section formula</u>

The coordinates of the point P(x, y) which divides the line segment joining the points A(x_1, y_1) and B(x_2, y_2), internally, in the ratio m_1 : m_2 are

$$\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

<u>Mid-point formula</u>

The coordinates of the point P(x, y) which is the midpoint of the line segment joining the points A(x_1, y_1) and B(x_2, y_2), are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

IMPORTANT QUESTIONS

Find the coordinates of the point which divides the line segment joining the points (4, – 3) and (8, 5) in the ratio 3 : 1 internally.

Solution : Let P(x, y) be the required point.

Using the section formula, $x=\frac{m_2x_1+m_1x_2}{m_1+m_2}, y=\frac{m_2y_1+m_1y_2}{m_1+m_2}$ we get

$$x=\frac{3(8)+1(4)}{3+1}=7, y=\frac{3(5)+1(-3)}{3+1}=3$$

Therefore, (7, 3) is the required point.

In what ratio does the point (– 4, 6) divide the line segment joining the points A(– 6, 10) and B(3, – 8)?

Solution : Let (– 4, 6) divide AB internally in the ratio k : 1.

Using the section formula, $x=\frac{m_2x_1+m_1x_2}{m_1+m_2}, y=\frac{m_2y_1+m_1y_2}{m_1+m_2}$ we get

$$y=\frac{k(-8)+1(10)}{k+1}=6$$

$$\Rightarrow -8k+10=6k+6 \Rightarrow -8k-6k=6-10$$

$$\Rightarrow -14k=-4 \Rightarrow k=\frac{4}{14}=\frac{2}{7}$$

Therefore, the point (– 4, 6) divides the line segment joining the points A(– 6, 10) and B(3, – 8) in the ratio 2 : 7.

Questions for practice

1. Find the coordinates of the point which divides the join of (–1, 7) and (4, –3) in the ratio 2 : 3.
2. Find the coordinates of the points of trisection of the line segment joining (4, –1) and (–2, –3).
3. Find the coordinates of the points of trisection (i.e., points dividing in three equal parts) of the line segment joining the points A(2, – 2) and B(– 7, 4).
4. Find the ratio in which the y-axis divides the line segment joining the points (5, – 6) and (–1, – 4). Also find the point of intersection.
5. Find the ratio in which the line segment joining the points (– 3, 10) and (6, – 8) is divided by (– 1, 6).
6. Find the ratio in which the line segment joining A(1, – 5) and B(– 4, 5) is divided by the x-axis. Also find the coordinates of the point of division.
7. Find the coordinates of the points which divide the line segment joining A(– 2, 2) and B(2, 8) into four equal parts.
8. If the points A(6, 1), B(8, 2), C(9, 4) and D(p, 3) are the vertices of a parallelogram, taken in order, find the value of p.
9. If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.
10. In what ratio does the x–axis divide the line segment joining the points (– 4, – 6) and (–1, 7)? Find the coordinates of the point of division.
11. If P (9a – 2, –b) divides line segment joining A (3a + 1, –3) and B (8a, 5) in the ratio 3 : 1, find the values of a and b.
12. If (a, b) is the mid-point of the line segment joining the points A (10, –6) and B (k, 4) and a – 2b = 18, find the value of k and the distance AB.
13. The centre of a circle is (2a, a – 7). Find the values of a if the circle passes through the point (11, –9) and has diameter $10\sqrt{2}$ units.
14. The line segment joining the points A (3, 2) and B (5,1) is divided at the point P in the ratio 1:2 and it lies on the line 3x – 18y + k = 0. Find the value of k.
15. Find the coordinates of the point R on the line segment joining the points P (–1, 3) and Q (2, 5) such that PR = $\frac{3}{5}$PQ.
16. Find the values of k if the points A (k + 1, 2k), B (3k, 2k + 3) and C (5k – 1, 5k) are collinear.
17. Find the ratio in which the line 2x + 3y – 5 = 0 divides the line segment joining the points (8, –9) and (2, 1). Also find the coordinates of the point of division.
18. The mid-points D, E, F of the sides of a triangle ABC are (3, 4), (8, 9) and (6, 7). Find the coordinates of the vertices of the triangle.

Area of a Triangle

If A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) are the vertices of a ΔABC, then the area of ΔABC is given by

$$Area\ \ of\ \ \Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

Trick to remember the formula

The formula of area of a triangle can be learn with the help of following arrow diagram:

$$\Delta ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix}$$

Find the sum of products of numbers at the ends of the lines pointing downwards and then subtract the sum of products of numbers at the ends of the line pointing upwards, multiply the difference by $\frac{1}{2}$. i.e. $Area\ \ of\ \ \Delta ABC = \frac{1}{2}[(x_1y_2 + x_2y_3 + x_3y_1) - (x_1y_3 + x_3y_2 + x_2y_1]$

IMPORTANT QUESTIONS

Find the area of a triangle whose vertices are (1, –1), (– 4, 6) and (–3, –5).

Solution: Here, A(1, –1), B(– 4, 6) and C (–3, –5).

Using the formula

$$\Delta ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_3 \\ x_1 & y_1 \end{vmatrix}$$

we get

$$\Delta ABC = \frac{1}{2}\begin{vmatrix} 1 & -1 \\ -4 & 6 \\ -3 & -5 \\ 1 & -1 \end{vmatrix}$$

$\Delta ABC = \frac{1}{2}[(6 + 20 + 3) - (-5 - 18 + 4)] = \frac{1}{2}[29 - (-19)] = \frac{1}{2}(29 + 19) = \frac{1}{2} \times 48 = 24$ sq. units

So, the area of the triangle is 24 square units.

Questions for practice

1. Find the area of a triangle formed by the points A(5, 2), B(4, 7) and C (7, – 4).

2. Find the area of the triangle formed by the points P(–1.5, 3), Q(6, –2) and R(–3, 4).

3. Find the value of k if the points A(2, 3), B(4, k) and C(6, –3) are collinear.

4. If A(–5, 7), B(– 4, –5), C(–1, –6) and D(4, 5) are the vertices of a quadrilateral, find the area of the quadrilateral ABCD.

5. Find the area of the triangle whose vertices are : (i) (2, 3), (–1, 0), (2, – 4) (ii) (–5, –1), (3, –5), (5, 2)

6. In each of the following find the value of ‘k’, for which the points are collinear. (i) (7, –2), (5, 1), (3, k) (ii) (8, 1), (k, – 4), (2, –5)

7. Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, –1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

8. Find the area of the quadrilateral whose vertices, taken in order, are (– 4, – 2), (– 3, – 5), (3, – 2) and (2, 3).

9. Find the value of m if the points (5, 1), (–2, –3) and (8, 2m) are collinear.

10. Find the area of the triangle whose vertices are (–8, 4), (–6, 6) and (–3, 9).

11. A (6, 1), B (8, 2) and C (9, 4) are three vertices of a parallelogram ABCD. If E is the midpoint of DC, find the area of Δ ADE.

12. The points A (2, 9), B (a, 5) and C (5, 5) are the vertices of a triangle ABC right angled at B. Find the values of a and hence the area of ΔABC.

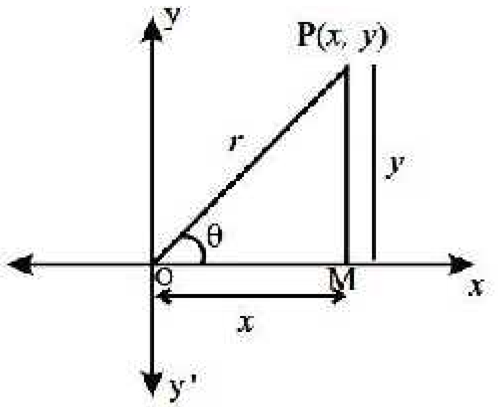
13. If the points A (1, –2), B (2, 3) C (a, 2) and D (– 4, –3) form a parallelogram, find the value of a and height of the parallelogram taking AB as base.

CHAPTER – 8 & 9
TRIGONOMETRY

Trigonometric Ratios (T - Ratios) of an acute angle of a right triangle

In XOY-plane, let a revolving line OP starting from OX, trace out $\angle XOP=\theta$. From P (x, y) draw PM $\perp$ to OX.

In right angled triangle OMP. OM = x (Adjacent side); PM = y (opposite side); OP = r (hypotenuse).



$$\sin\theta = \frac{\text{Opposite Side}}{\text{Hypotenuse}} = \frac{y}{r}, \quad \cos\theta = \frac{\text{Adjacent Side}}{\text{Hypotenuse}} = \frac{x}{r}, \quad \tan\theta = \frac{\text{Opposite Side}}{\text{Adjacent Side}} = \frac{y}{x}$$

$$\operatorname{cosec}\theta = \frac{\text{Hypotenuse}}{\text{Opposite Side}} = \frac{r}{y}, \quad \sec\theta = \frac{\text{Hypotenuse}}{\text{Adjacent Side}} = \frac{r}{x}, \quad \cot\theta = \frac{\text{Adjacent Side}}{\text{Opposite Side}} = \frac{x}{y}$$

Reciprocal Relations

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}, \ \sec\theta = \frac{1}{\cos\theta} \ \textbf{and} \ \cot\theta = \frac{1}{\tan\theta}$$

Quotient Relations

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \ \text{and} \ \cot\theta = \frac{\cos\theta}{\sin\theta}$$

IMPORTANT QUESTIONS

If $\tan A = \frac{4}{3}$ **, find the value of all T– ratios of** θ**.**

Solution: Given that, In right Δ ABC, $\tan A = \frac{BC}{AB} = \frac{4}{3}$

Therefore, if BC = 4k, then AB = 3k, where k is a positive number.

Now, by using the Pythagoras Theorem, we have

$AC^2 = AB^2 + BC^2 = (4k)^2 + (3k)^2 = 25k^2$

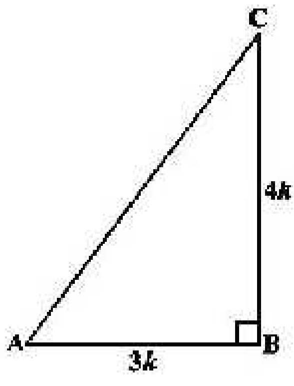
So, AC = 5k

Now, we can write all the trigonometric ratios using their definitions

$$\sin A = \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5}, \ \cos A = \frac{AB}{AC} = \frac{3k}{5k} = \frac{3}{5}$$

and $\cot A = \frac{1}{\tan A} = \frac{3}{4}$,

$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{5}{4}$,



$\sec A = \frac{1}{\cos A} = \frac{5}{3}$

Questions for Practice

1. If $\sin\theta = \frac{5}{13}$, find the value of all T– ratios of θ.

2. If $\cos\theta = \frac{7}{25}$, find the value of all T– ratios of θ.

3. If $\tan\theta = \frac{15}{8}$, find the value of all T– ratios of θ.

4. If $\cot\theta = 2$, find the value of all T– ratios of θ.

5. If $\text{cosec}\,\theta = \sqrt{10}$, find the value of all T– ratios of θ.

6. In Δ OPQ, right-angled at P, OP = 7 cm and OQ – PQ = 1 cm. Determine the values of sin Q and cos Q.

7. In Δ PQR, right-angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

Trigonometric ratios for angle of measure.
0^0, 30^0,45^0, 60^0 and 90^0 in tabular form.

$\angle$**A**	**0^0**	**30^0**	**45^0**	**60^0**	**90^0**
sinA	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cosA	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tanA	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosecA	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
secA	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cotA	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

IMPORTANT QUESTIONS

If cos (A – B) = $\frac{\sqrt{3}}{2}$ and sin (A + B) = 1, then find the value of A and B.

Solution: Given that $\cos(A-B) = \frac{\sqrt{3}}{2} = \cos 30^0$

$\Rightarrow A - B = 30^0$ ………………. (1)

and $\sin(A+B) = 1 = \sin 90^0$

$\Rightarrow A + B = 90^0$ …………………… (2)

Solving equations (1) and (2), we get A = 60^0 and B = 30^0.

Questions for Practice

Evaluate each of the following:

1. $\sin 60^0 \cos 30^0 + \cos 60^0 \sin 30^0$

2. $\cos 60^0 \cos 30^0 - \sin 60^0 \sin 30^0$

3. $\cos 45^0 \cos 30^0 + \sin 45^0 \sin 30^0$

4. $\sin 60^0 \sin 45^0 - \cos 60^0 \cos 45^0$

5. $(\sin^2 30^0 + 4\cot^2 45^0 - \sec^2 60^0)(\cos ec^2 45^0 \sec^2 30^0)$

6. If sin (A – B) = $\frac{1}{2}$ and cos(A + B) = $\frac{1}{2}$, then find the value of A and B.

7. If tan (A – B) = $\frac{1}{\sqrt{3}}$ and tan (A + B) = $\sqrt{3}$, then find the value of A and B.

Trigonometric ratios of Complementary angles.

sin (90 – θ) = cos θ | cos (90 – θ) = sin θ

tan (90 – θ) = cot θ | cot (90 – θ) = tan θ

sec (90 – θ) = cosec θ | cosec (90 – θ) = sec θ.

IMPORTANT QUESTIONS

If sin 3A = cos (A – 26°), where 3A is an acute angle, find the value of A.

Solution: Given that sin 3A = cos (A – 26°). (1)

Since sin 3A = cos (90° – 3A), we can write (1) as

cos (90° – 3A) = cos (A – 26°)

Since 90° – 3A and A – 26° are both acute angles, therefore comparing both sides we get,

90° – 3A = A – 26° which gives A = 29°

Questions for Practice

1. Express cot 85° + cos 75° in terms of trigonometric ratios of angles between 0° and 45°.

2. Express sin 67° + cos 75° in terms of trigonometric ratios of angles between 0° and 45°.

3. If tan 2A = cot (A – 18°), where 2A is an acute angle, find the value of A.

4. If tan A = cot B, prove that A + B = 90°.

5. If sec 4A = cosec (A – 20°), where 4A is an acute angle, find the value of A.

6. If A, B and C are interior angles of a triangle ABC, then show that

TRIGONOMETRIC IDENTITIES

An equation involving trigonometric ratios of an angle is said to be a trigonometric identity if it is satisfied for all values of θ for which the given trigonometric ratios are defined.

Identity (1) : $\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \sin^2\theta = 1 - \cos^2\theta$ and $\cos^2\theta = 1 - \sin^2\theta$.

Identity (2) : $\sec^2\theta = 1 + \tan^2\theta$

$\Rightarrow \sec^2\theta - \tan^2\theta = 1$ and $\tan^2\theta = \sec^2\theta - 1$.

Identity (3) : $\text{cosec}^2\theta = 1 + \cot^2\theta$

$\Rightarrow \text{cosec}^2\theta - \cot^2\theta = 1$ and $\cot^2\theta = \text{cosec}^2\theta - 1$.

IMPORTANT QUESTIONS

Prove that: $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \cos ecA + \cot A$

Solution: LHS = $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

(Dividing Numerator and Denominator by sinA, we get)

$$= \frac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} - \frac{1}{\sin A}} = \frac{\cot A - 1 + \operatorname{cos}ecA}{\cot A + 1 - \operatorname{cos}ecA} \qquad \left[\because \cot A = \frac{\cos A}{\sin A}, \operatorname{cos}ecA = \frac{1}{\sin A}\right]$$

$$= \frac{\cot A + \operatorname{cos}ecA - 1}{\cot A + 1 - \operatorname{cos}ecA} = \frac{\cot A + \operatorname{cos}ecA - (\operatorname{cos}ec^2 A - \cot^2 A)}{\cot A + 1 - \operatorname{cos}ecA} \qquad [\because \operatorname{cos}ec^2 A - \cot^2 A = 1]$$

$$= \frac{\cot A + \operatorname{cos}ecA - (\operatorname{cos}ecA + \cot A)(\operatorname{cos}ecA - \cot A)}{\cot A + 1 - \operatorname{cos}ecA}$$

$$= \frac{(\operatorname{cos}ecA + \cot A)(1 - \operatorname{cos}ecA + \cot A)}{\cot A + 1 - \operatorname{cos}ecA} = \operatorname{cos}ecA + \cot A = RHS$$

Questions for Practice

Prove the following identities:

1. sec A (1 – sin A)(sec A + tan A) = 1.

2. $\dfrac{\cot A - \cos A}{\cot A + \cos A} = \dfrac{\operatorname{cos}ecA - 1}{\operatorname{cos}ecA + 1}$

3. $\dfrac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \dfrac{1}{\sec\theta - \tan\theta}$

4. $(\operatorname{cos}ec\theta - \cot\theta)^2 = \dfrac{1 - \cos\theta}{1 + \cos\theta}$

5. $\dfrac{\cos A}{1 + \sin A} + \dfrac{1 + \sin A}{\cos A} = 2\sec A$

6. $\dfrac{\tan\theta}{1 - \cot\theta} + \dfrac{\cot\theta}{1 - \tan\theta} = 1 + \sec\theta \operatorname{cos}ec\theta$

7. $\dfrac{1 + \sec A}{\sec A} = \dfrac{\sin^2 A}{1 - \cos A}$

8. $\sqrt{\dfrac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$

9. $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

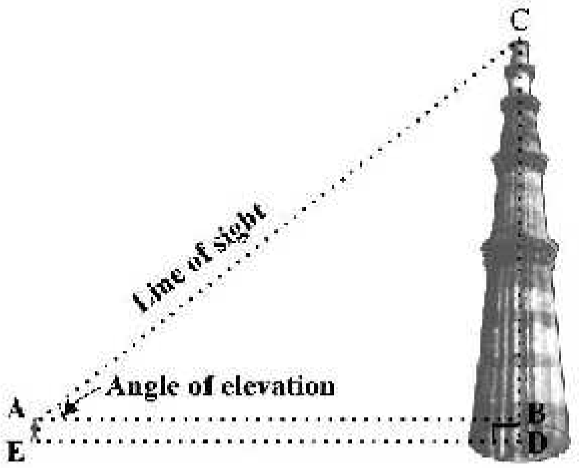
10. $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

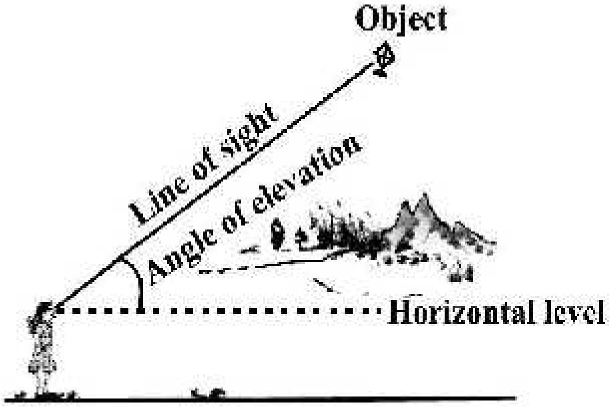
11. $(\operatorname{cos}ecA - \sin A)(\sec A - \cos A) = \dfrac{1}{\tan A + \cot A}$

12. $\left(\dfrac{1 + \tan^2 A}{1 + \cot^2 A}\right) = \left(\dfrac{1 - \tan A}{1 - \cot A}\right)^2 = \tan^2 A$

<u>ANGLE OF ELEVATION</u>

In the below figure, the line AC drawn from the eye of the student to the top of the minar is called the *line of sight*. The student is looking at the top of the minar. The angle BAC, so formed by the line of sight with the horizontal, is called the *angle of elevation* of the top of the minar from the eye of the student. Thus, the **line of sight** is the line drawn from the eye of an observer to the point in the object viewed by the observer.

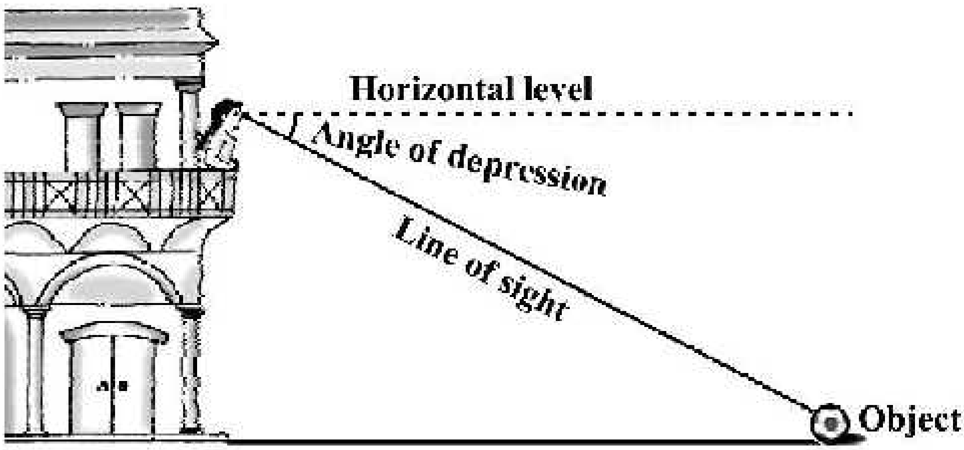




The **angle of elevation** of the point viewed is the angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level, i.e., the case when we raise our head to look at the object

ANGLE OF DEPRESSION

In the below figure, the girl sitting on the balcony is *looking down* at a flower pot placed on a stair of the temple. In this case, the line of sight is *below* the horizontal level. The angle so formed by the line of sight with the horizontal is called the *angle of depression*. Thus, the **angle of depression** of a point on the object being viewed is the angle formed by the line of sight with the horizontal when the point is below the horizontal level, i.e., the case when we lower our head to look at the point being viewed



IMPORTANT QUESTIONS

The angles of depression of the top and the bottom of an 8 m tall building from the top of a multi-storeyed building are 30° and 45°, respectively. Find the height of the multi-storeyed building and the distance between the two buildings.

Solution : Let PC = h m be the height of multistoryed building and AB denotes the 8 m tall building.

BD = AC = x m, PC = h = PD + DC = PD + AB = PD + 8 m

So, PD = h – 8 m

Now, $\angle$QPB = $\angle$PBD = 30°

Similarly, $\angle$QPA = $\angle$PAC = 45°.

In right Δ PBD, $\tan 30^0 = \frac{PD}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h-8}{x}$

$\Rightarrow x = (h-8)\sqrt{3}\ \ m$ ………………. (1)

Also, In right Δ PAC, $\tan 45^0 = \frac{PC}{AC} \Rightarrow 1 = \frac{h}{x}$

$\Rightarrow x = h\ m$ ………………….. (2)

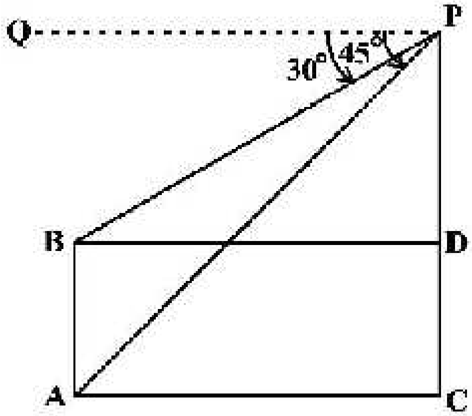
From equations (1) and (2), we get $h = (h-8)\sqrt{3}$

$\Rightarrow h = h\sqrt{3} - 8\sqrt{3} \Rightarrow h\sqrt{3} - h = 8\sqrt{3}$

$\Rightarrow h(\sqrt{3}-1) = 8\sqrt{3} \Rightarrow h = \frac{8\sqrt{3}}{\sqrt{3}-1}$

$\Rightarrow h = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{8\sqrt{3}(\sqrt{3}+1)}{3-1}$

$\Rightarrow h = \frac{8(3+\sqrt{3})}{2} = 4(3+\sqrt{3})m$



Hence, the height of the multi-storeyed building is $4(3+\sqrt{3})m$ and the distance between the two buildings is also $4(3+\sqrt{3})m$.

From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45°, respectively. If the bridge is at a height of 3 m from the banks, find the width of the river.

Solution: Let A and B represent points on the bank on opposite sides of the river, so that AB is the width of the river. P is a point on the bridge at a height of 3 m, i.e., DP = 3 m.

Now, AB = AD + DB

In right Δ APD, $\tan 30^0 = \frac{PD}{AD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{3}{AD}$

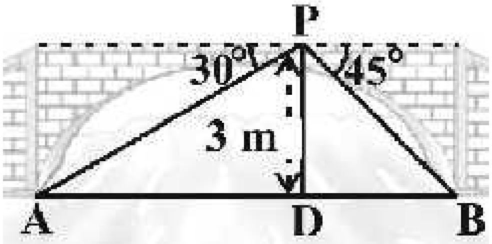
$\Rightarrow AD = 3\sqrt{3}\ m$

Also, in right Δ PBD, $\tan 45^0 = \frac{PD}{BD} \Rightarrow 1 = \frac{3}{BD}$

$\Rightarrow BD = 3\ m$

Now, AB = BD + AD = $3 + 3\sqrt{3} = 3(1+\sqrt{3})\ m$

Therefore, the width of the river is $3(1+\sqrt{3})\ m$



Questions for Practice

1. The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.
2. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.
3. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.
4. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.
5. A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.
6. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

7. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles and the distances of the point from the poles.

8. A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower, the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joing this point to the foot of the tower, the angle of elevation of the top of the tower is 30° . Find the height of the tower and the width of the canal.

9. From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

10. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

11. A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30° . Find the distance travelled by the balloon during the interval.

12. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

13. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

CHAPTER – 13
SURFACE AREAS AND VOLUMES

IMPORTANT FORMULAE

Name of the Solid	Curved Surface Area	Total Surface Area	Volume
Cuboid	$2h(l+b)$	$2(lb+bh+hl)$	lbh
Cube	$4a^2$	$6a^2$	a^3
Right Circular Cylinder	$2\pi rh$	$2\pi r(r+h)$	$\pi r^2 h$
Right Circular Cone	πrl	$2\pi r(r+l)$	$\frac{1}{3}\pi r^2 h$
Sphere	–	$4\pi r^2$	$\frac{4}{3}\pi r^3$
Hemisphere	$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$
Frustum of a Cone	$\pi(r_1+r_2)l$ where $l=\sqrt{h^2+(r_1-r_2)^2}$	$\pi(r_1+r_2)l + \pi r_1^2 + \pi r_2^2$	$\frac{1}{3}\pi h(r_1^2+r_2^2+r_1r_2)$

COMBINATIONAL FIGURE BASED QUESTIONS

IMPORTANT QUESTIONS

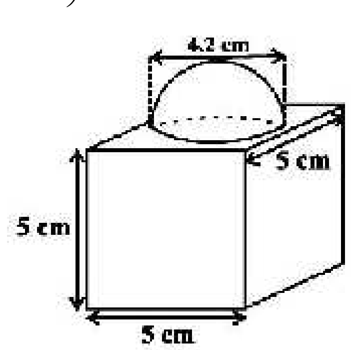
The decorative block is shown in below left figure made of two solids — a cube and a hemisphere. The base of the block is a cube with edge 5 cm, and the hemisphere fixed on the top has a diameter of 4.2 cm. Find the total surface area of the block.

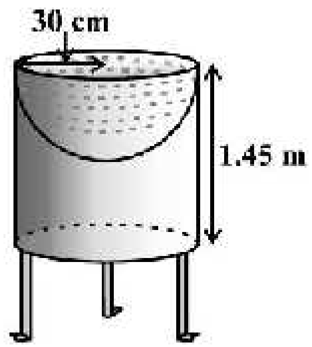
Solution: The total surface area of the cube = 6 × (edge)^2 = 6 × 5 × 5 cm^2 = 150 cm^2.

So, the surface area of the block = TSA of cube – base area of hemisphere + CSA of hemisphere

= 150 – πr^2 + $2\pi r^2$ = (150 + πr^2) cm2

$$=150+\left(\frac{22}{7}\times\frac{4.2}{2}\times\frac{4.2}{2}\right)cm^2 =150+13.86cm^2 =163.86cm^2$$





Mayank made a bird-bath for his garden in the shape of a cylinder with a hemispherical depression at one end. The height of the cylinder is 1.45 m and its radius is 30 cm. Find the total surface area of the bird-bath.

Solution : Let h be height of the cylinder, and r the common radius of the cylinder and hemisphere. (See above right sided figure)

Total surface area of the bird-bath = CSA of cylinder + CSA of hemisphere

= 2πrh + 2πr2 = 2π r (h + r) $= 2\times\frac{22}{7}\times 30(145+30) = 2\times\frac{22}{7}\times 30\times 175$ $= 33000cm^2 = 3.3m^2$

A juice seller was serving his customers using glasses as shown in below figure. The inner diameter of the cylindrical glass was 5 cm, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of a glass was 10 cm, find the apparent capacity of the glass and its actual capacity. (Use π = 3.14.)

Solution: Here, inner diameter = 5 cm. height, h = 10 cm

So, radius, r = $\frac{5}{2}$ cm

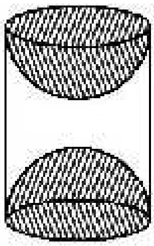
Apparent capacity of the glass = Volume of cylinder – Volume of hemisphere

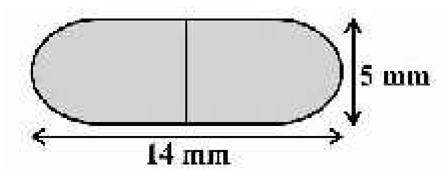
$$=\pi r^2 h - \frac{2}{3}\pi r^3 = \pi r^2\left(h - \frac{2}{3}r\right) = 3.14\times\frac{5}{2}\times\frac{5}{2}\times\left(0 - \frac{2}{3}\times\frac{5}{2}\right)$$

$$= 3.14\times\frac{25}{4}\times\frac{25}{3} = \frac{19625}{12} = 163.54cm^3$$

Questions for Practice

1. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder (see below left figure). If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article.





2. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (see above right sided figure). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.

3. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of Rs 500 per m^2.

4. From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.

5. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

6. A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy. If a right circular cylinder circumscribes the toy, find the difference of the volumes of the cylinder and the toy. (Take π = 3.14)

7. A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm

8. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm3 of iron has approximately 8g mass. (Use π = 3.14)

9. A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

CONVERSION BASED QUESTIONS

IMPORTANT QUESTIONS

A cone of height 24 cm and radius of base 6 cm is made up of modelling clay. A child reshapes it in the form of a sphere. Find the radius of the sphere.

Solution: Here, radius of cone, r = 6 cm, height of cone, h = 24 cm

Let the radius of the sphere be R cm, then we have

Volume of Sphere = Volume of cone

$$\Rightarrow \frac{4}{3}\text{®}R^3 = \frac{1}{3}\text{®}r^2h \Rightarrow 4R^3 = r^2h \Rightarrow R^3 = \frac{r^2h}{4} = \frac{6\times 6\times 24}{4} = 6\times 6\times 6 \Rightarrow R = 6cm$$

Therefore, the radius of the sphere is 6 cm.

Questions for Practice

1. A copper rod of diameter 1 cm and length 8 cm is drawn into a wire of length 18 m of uniform thickness. Find the thickness of the wire.

2. A hemispherical tank full of water is emptied by a pipe at the rate of litres per second. How much time will it take to empty half the tank, if it is 3m in diameter?

3. A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.

4. A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.

5. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed?

6. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in her field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled?

FRUSTUM OF A CONE BASED QUESTIONS

IMPORTANT QUESTIONS

A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass.

Solution: Here, height of frustum of cone, h = 14 cm, diameters of its two circular ends are 4 cm and 2 cm

So, radii of its two circular ends are R = 2 cm and r = 1 cm

Now, Capacity of the glass = Volume of a frustum of a cone

$$= \frac{\text{®}h}{3}(R^2 + r^2 + Rr) = \frac{22}{7}\times\frac{14}{3}(2^2 + 1^2 + 2\times 1) = \frac{44}{3}(4+1+2) \quad = \frac{44}{3}\times 7 = \frac{308}{3} = 102.67cm^3$$

The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

Solution: Here, slant height of a frustum of a cone, l = 4 cm,

Circumference of upper end = 2πr = 6 cm

So, πr = 3 cm

and Circumference of upper end = 2πR = 18 cm

So, πR = 9 cm

Now, curved surface area of the frustum = πl(R + r) = l x (πR + πr)

= 4 x (9 + 3) = 4 x 12 = 48cm^2

Questions for Practice

1. The radii of the ends of a frustum of a cone 45 cm high are 28 cm and 7 cm. Find its volume, the curved surface area and the total surface area
2. An open metal bucket is in the shape of a frustum of a cone, mounted on a hollow cylindrical base made of the same metallic sheet. The diameters of the two circular ends of the bucket are 45 cm and 25 cm, the total vertical height of the bucket is 40 cm and that of the cylindrical base is 6 cm. Find the area of the metallic sheet used to make the bucket, where we do not take into account the handle of the bucket. Also, find the volume of water the bucket can hold.
3. A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container, at the rate of Rs 20 per litre. Also find the cost of metal sheet used to make the container, if it costs Rs 8 per 100 cm2. (Take π = 3.14)
4. A metallic right circular cone 20 cm high and whose vertical angle is 60° is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter cm, find the length of the wire.

SAMPLE PAPER 01 (2018-19)

SUBJECT: MATHEMATICS **MAX. MARKS : 80**
CLASS : X **DURATION : 3 HRS**

General Instruction:

(i) **All** questions are compulsory.
(ii) This question paper contains **30** questions divided into four Sections A, B, C and D.
(iii) **Section A** comprises of 6 questions of **1 mark** each. **Section B** comprises of 6 questions of **2 marks** each. **Section C** comprises of 10 questions of **3 marks** each and **Section D** comprises of 8 questions of **4 marks** each.
(iv) There is no overall choice. However, an internal choice has been provided in two questions in 1 mark each, two questions in 2 marks each, four questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of Calculators is not permitted

SECTION – A

Questions 1 to 6 carry 1 mark each.

1. ABC and BDE are two equilateral triangles such that $BD = \frac{2}{3}BC$. Find the ratio of the areas of triangles ABC and BDE.

2. Find the coordinates of the point on y-axis which is nearest to the point (–2, 5).

3. If sinA = $\frac{1}{2}$, find the value of $\frac{2\sec A}{1+\tan^2 A}$.

OR

If $\sin^{-} = \cos^{-}$, then find the value of $2\tan^{-} + \cos^{2-}$

4. Find the values of k for quadratic equation $2x^2 + kx + 3 = 0$, so that they have two equal roots.

OR

Find the value of k, for which one root of the quadratic equation $kx^2 – 14x + 8 = 0$ is 2.

5. Express 7429 as a product of its prime factors.

6. Which term of the AP: 3, 8, 13, 18 ,…. Is 78?

SECTION – B

Questions 6 to 12 carry 2 marks each.

7. A box contains cards numbered 11 to 123. A card is drawn at random from the box. Find the probability that the number on the drawn card is (i) a square number (ii) a multiple of 7

8. A box contains 12 balls of which some are red in colour. If 6 more red balls are put in the box and a ball is drawn at random, the probability of drawing a red ball doubles than what it was before. Find the number of red balls in the bag.

9. Prove that $5-2\sqrt{3}$ is an irrational number.

OR

The HCF and LCM of two numbers are 9 and 360 respectively. If one number is 45, find the other number.

10. Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of 'm' for which $y = mx + 3$.

11. Find the ratio in which the line segment joining the points (– 3, 10) and (6, – 8) is divided by (–1, 6).

12. The sum of first n terms of an AP is given by $S_n = 2n^2 + 3n$. Find the sixteenth term of the AP.

OR

Find the 20th term from the last term of the AP 3,8,13,….,253

SECTION – C

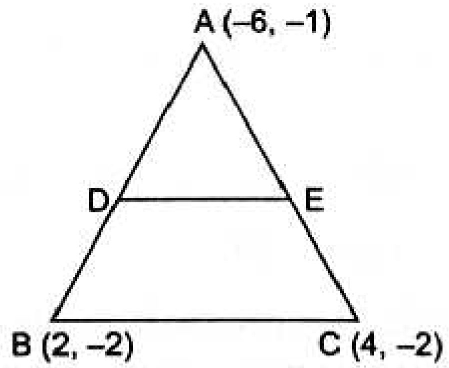
Questions 13 to 22 carry 3 marks each.

13. Using Euclid's division algorithm, find the HCF of 2160 and 3520.

14. Four points A(6, 3), B(-3, 5), C(4, -2) and D(x, 3x) are given such that $\frac{\Delta DBC}{\Delta ABC} = \frac{1}{2}$, find x.

OR

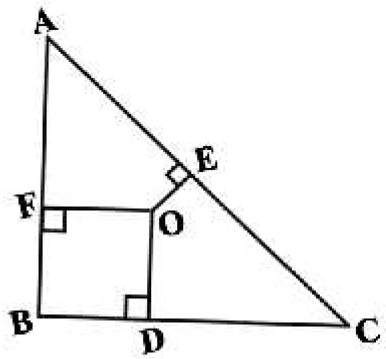
In the given below figure, in ΔABC, D and E are the midpoint of the sides BC and AC respectively. Find the length of DE. Prove that DE = $\frac{1}{2}$AB



15. In the below figure, O is a point in the interior of a triangle ABC, OD ⊥ BC, OE ⊥ AC and OF ⊥ AB. Show that

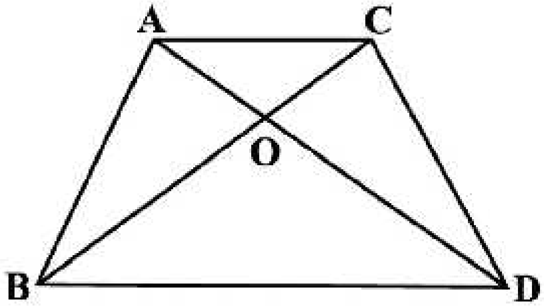
(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$,

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$.



OR

In the below figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$

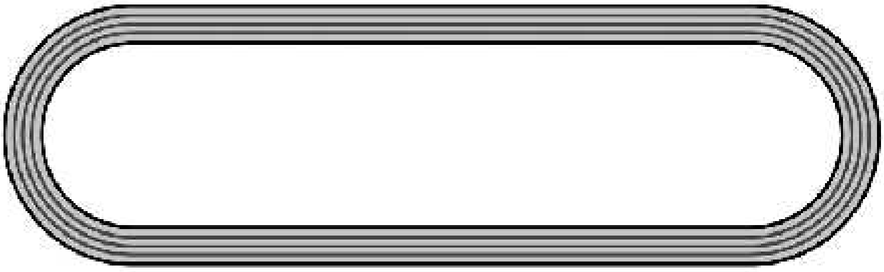


16. Evaluate without using tables: $\dfrac{\sec\theta\operatorname{cosec}(90^0-\theta)-\tan\theta\cot(90^0-\theta)+(\sin^2 35^0+\sin^2 55^0)}{\tan 10^0\tan 20^0\tan 45^0\tan 70^0\tan 80^0}$

OR

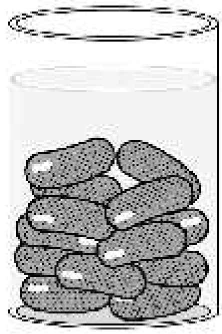
Prove that: $\dfrac{1}{\operatorname{cosec}A-\cot A}-\dfrac{1}{\sin A}=\dfrac{1}{\sin A}-\dfrac{1}{\operatorname{cosec}A+\cot A}$.

17. The below figure depicts a racing track whose left and right ends are semicircular.



The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find :
(i) the distance around the track along its inner edge
(ii) the area of the track.

18. A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm (see below figure).

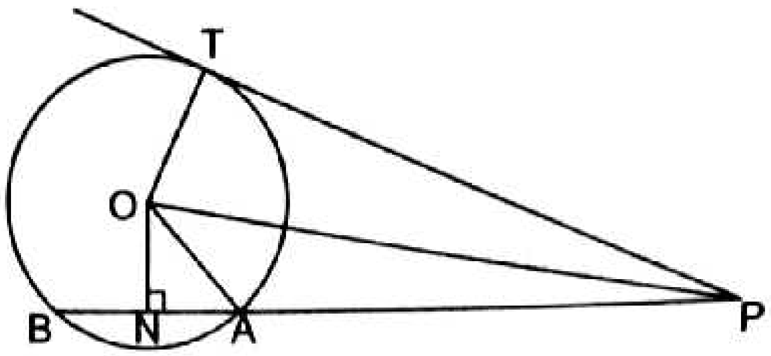


OR

A cone of maximum size is carved out from a cube of edge 14 cm. Find the surface area of the remaining solid after the cone is carved out.

19. Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

20. In the given below figure, from an external point P, a tangent PT and a secant PAB is drawn to a circle with centre O. ON is perpendicular on the chord AB. Prove that PA. PB = PT^2.



21. If two zeroes of the polynomial $x^4 + 3x^3 - 20x^2 - 6x + 36$ are $\sqrt{2}$ and $-\sqrt{2}$, find the other zeroes of the polynomial.

22. Find the mode of the following frequency distribution:

Marks	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Number of students	15	30	45	12	18

SECTION – D

Questions 23 to 30 carry 4 marks each.

23. An aeroplane at an altitude of 300 m observes the angles of depression of opposite points on the two banks of a river to be 45^0 and 60^0. Find the width of the river.

OR

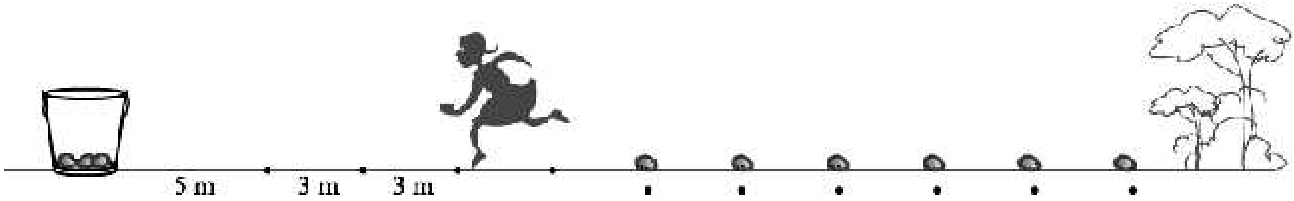
A man on the top of a vertical observation tower observes a car moving at a uniform speed coming directly towards it. If it takes 12 minutes for the angle of depression to change from 30^0 to 45^0, how long will the car take to reach the observation tower from this point?

24. A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

OR

An aeroplane left 40 minutes late due to heavy rains and in order to reach its destination, 1600 km away in time, it had to increase its speed by 400 km/hour from its original speed. Find the original speed of the aeroplane.

25. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato, and the other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line (see below figure).



A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?

26. Prove that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

27. Akshay took a right circular cylinder having base diameter 12cm and height 15cm and filled it completely with ice-cream. He then went to a slum area and distributed the ice-cream filled in cones of height 12cm and diameter 6cm each having a hemispherical shape on the top to the needy children. Find the number of children who will get ice cream in these cones. What are the values of Akshay that are depicted here ?

28. Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameter each at a distance of 7 cm from its centre. Draw tangents to the circle from these two points P and Q.

29. Prove that $\frac{\sin\,\bar{} - \cos\,\bar{} + 1}{\sin\,\bar{} + \cos\,\bar{} - 1} = \frac{1}{\sec\,\bar{} - \tan\,\bar{}}$

30. If the median of the distribution given below is 28.5, find the values of *x* and *y*.

C. I.	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	Total
F	5	x	20	15	y	5	100

OR

Draw more than ogive for the following frequency distribution:

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Number of students	5	8	6	10	6	6

Also find the median from the graph and verify that by using the formula.

SAMPLE PAPER 02 (2018-19)

SUBJECT: MATHEMATICS | **MAX. MARKS : 80**
CLASS : X | **DURATION : 3 HRS**

General Instruction:

(i) **All** questions are compulsory.
(ii) This question paper contains **30** questions divided into four Sections A, B, C and D.
(iii) **Section A** comprises of 6 questions of **1 mark** each. **Section B** comprises of 6 questions of **2 marks** each. **Section C** comprises of 10 questions of **3 marks** each and **Section D** comprises of 8 questions of **4 marks** each.
(iv) There is no overall choice. However, an internal choice has been provided in two questions in 1 mark each, two questions in 2 marks each, four questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of Calculators is not permitted

SECTION – A

Questions 1 to 6 carry 1 mark each.

1. For what value of k will k + 9, 2k – 1 and 2k + 7 are the consecutive terms of an A.P.?

2. If product of two numbers is 3691 and their LCM is 3691, find their HCF.

3. If –5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots, find the value of k.

OR

Find the value(s) of k for which the equation $x^2 + 5kx + 16 = 0$ has real and equal roots.

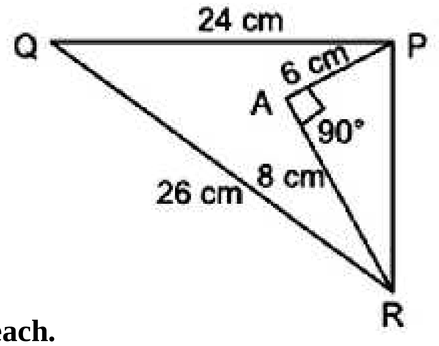
4. Find a relation between *x* and *y* if the points (*x*, *y*), (1, 2) and (7, 0) are collinear.

5. If A, B and C are the interior angles of triangle ABC, find $\tan\left(\frac{B+C}{2}\right)$

OR

Write the value of $\cot^2 - \frac{1}{\sin^2 -}$

6. In the adjoining figure, PQ = 24 cm, QR = 26 cm, PAR = 90°, PA = 6 cm and AR = 8 cm. Find QPR.



SECTION – B

Questions 6 to 12 carry 2 marks each.

7. Let P and Q be the points of trisection of the line segment joining the points A(2, –2) and B(–7, 4) such that P is nearer to A. Find the coordinates of P and Q.

8. In a morning walk, three persons step off together and their steps measure 40 cm, 42 cm and 45 cm, respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps?

OR

Prove that $7 - \sqrt{5}$ is an irrational number.

9. Three different coins are tossed together. Find the probability of getting (i) exactly two heads (ii) at least two heads

10. A card is drawn at random from a well-shuffled pack of 52 playing cards. Find the probability of getting (i) neither a red card nor a queen (ii) a face card or a spade card.

11. For what value of k, the following pair of linear equations has infinite number of solutions:
$kx+3y=(2k+1);\quad 2(k+1)x+9y=(7k+1).$

12. If the ratio of the sum of first n terms of two A.P's is (7n + 1) : (4n + 27), find the ratio of their 10th terms.

OR

If 7 times the 7th term of an A.P is equal to 11 times its 11th term, then find its 18th term.

SECTION – C

Questions 13 to 22 carry 3 marks each.

13. Use Euclid's Division Algorithm to find the HCF of 726 and 275.

14. Obtain all the zeroes of $3x^4+6x^3-2x^2-10x+5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

15. The perpendicular from A on side BC of a ΔABC intersects BC at D such that DB = 3 CD. Prove that $2AB^2 = 2AC^2 + BC^2$.

OR

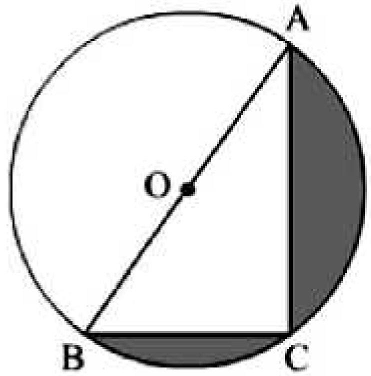
In an equilateral triangle ABC, D is a point on side BC such that BD = $\frac{1}{3}$BC. Prove that $9AD^2 = 7AB^2$.

16. A conical vessel, with base radius 5 cm and height 24 cm, is full of water. This water is emptied into a cylindrical vessel of base radius 10 cm. Find the height to which the water will rise in the cylindrical vessel. [Use π=22/7]

OR

A sphere of diameter 12 cm, is dropped in a right circular cylindrical vessel, partly filled with water. If the sphere is completely submerged in water, the water level in the cylindrical vessel rises by $3\frac{5}{9}$ cm. Find the diameter of the cylindrical vessel.

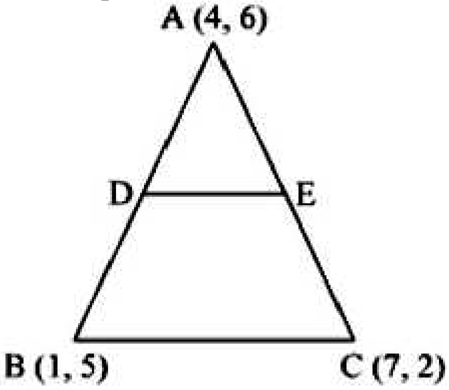
17. In the below figure, O is the centre of a circle such that diameter AB = 13 cm and AC = 12 cm. BC is joined. Find the area of the shaded region. (Take π = 3.14)



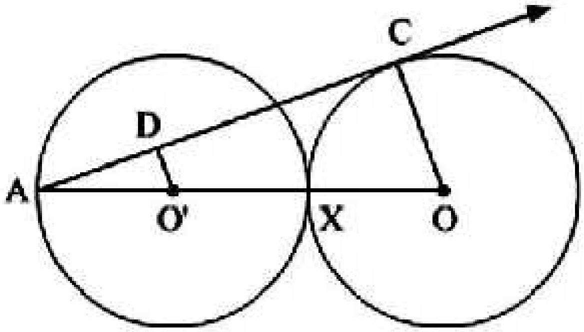
18. If the point P(x, y) is equidistant from the points A(a + b, b − a) and B(a − b, a + b). Prove that bx = ay.

OR

In the below figure, the vertices of ΔABC are A(4, 6), B(1, 5) and C(7, 2). A line-segment DE is drawn to intersect the sides AB and AC at D and E, respectively, such that $\frac{AD}{AB}=\frac{AE}{AC}=\frac{1}{3}$. Calculate the area of ΔADE and compare it with area of ΔABC.



19. In the below figure, two equal circles, with centres O and O', touch each other at X.OO' produced meets the circle with centre O' at A. AC is tangent to the circle with centre O, at the point C. O'D is perpendicular to AC. Find the value of DO'/CO.



20. Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

21. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, prove that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

OR

Evaluate: $\frac{2\sin 68^0}{\cos 22^0} - \frac{2\cot 15^0}{5\tan 75^0} - \frac{3\tan 45^0 \tan 20^0 \tan 40^0 \tan 50^0 \tan 70^0}{5}$

22. Find the mode age of the patients from the following distribution :

Age(in years)	6-15	16-25	26-35	36-45	46-55	56-65
No. of patients	6	11	21	23	14	5

SECTION – D

Questions 23 to 30 carry 4 marks each.

23. The houses in a row are numbered consecutively from 1 to 49. Show that there exists a value of X such that sum of numbers of houses proceeding the house numbered X is equal to sum of the numbers of houses following X.

24. Draw a circle of radius 4 cm. Draw two tangents to the circle inclined at an angle of 60° to each other.

25. The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60°. From a point Y, 40 m vertically above X, the angle of elevation of the top Q of tower is 45°. Find the height of the tower PQ and the distance PX. (Use $\sqrt{3}$=1.73).

OR

The angle of elevation of a cloud from a point 60 m above the surface of the water of a lake is 30^0 and the angle of depression of its shadow from the same point in water of lake is 60^0. Find the height of the cloud from the surface of water.

26. Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively offered to the state government to provide place and the canvas for 1500 tents to be fixed by the government and decided to share the whole expenditure equally. The lower part of each tent is cylindrical of base radius 2.8 m and height 3.5 m, with conical upper part of same base radius but of height 2.1 m. If the canvas used to make the tents costs Rs 120 per sq.m, find the amount shared by each school to set up the tents. What value is generated by the above problem? [Use π=22/7]

27. Solve for x : $\dfrac{1}{(x-1)(x-2)}+\dfrac{1}{(x-2)(x-3)}=\dfrac{2}{3}, x\neq 1,2,3$

OR

Solve for x : $\dfrac{1}{x+1}+\dfrac{2}{x+2}=\dfrac{4}{x+4}, x\neq -1,-2,-4$

28. Prove that "The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides."

29. Prove that : $\dfrac{\tan A}{1-\cot A}+\dfrac{\cot A}{1-\tan A}=1+\sec A\,\mathrm{cosec}A$.

30. Find the missing frequencies f_1 and f_2 in table given below; it is being given that the mean of the given frequency distribution is 50.

Class	0-20	20-40	40-60	60-80	80-100	Total
Frequency	17	f_1	32	f_2	19	120

OR

For the following distribution, draw the cumulative frequency curve more than type and hence obtain the median from the graph.

Marks	Below 10	Below 20	Below 30	Below 40	Below 50	Below 60
No. of Students	6	15	29	41	60	70

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SAMPLE PAPER 03 (2018-19)

SUBJECT: MATHEMATICS **MAX. MARKS : 80**
CLASS : X **DURATION : 3 HRS**

General Instruction:

(i) **All** questions are compulsory.
(ii) This question paper contains **30** questions divided into four Sections A, B, C and D.
(iii) **Section A** comprises of 6 questions of **1 mark** each. **Section B** comprises of 6 questions of **2 marks** each. **Section C** comprises of 10 questions of **3 marks** each and **Section D** comprises of 8 questions of **4 marks** each.
(iv) There is no overall choice. However, an internal choice has been provided in two questions in 1 mark each, two questions in 2 marks each, four questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of Calculators is not permitted

SECTION – A

Questions 1 to 6 carry 1 mark each.

1. Find the next term of the A.P. $\sqrt{7}, \sqrt{28}, \sqrt{63}, \ldots\ldots..$

2. Without actually performing the long division, find if $\frac{987}{10500}$ will have terminating or non-terminating (repeating) decimal expansion. Give reasons for your answer.

3. If 2 is a root of the quadratic equation $3x^2 + px - 8 = 0$ and the quadratic equation $4x^2 - 2px + k = 0$ has equal roots, find the value of k.

OR

For what value of k, are the roots of the quadratic equation $3x^2 + 2kx + 27 = 0$ real and equal.

4. Find the perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0).

5. If $\sin A = \frac{24}{25}$, then find the value of cosA.

OR

If $\tan \theta = \cot (30° + \theta)$, find the value of θ.

6. Sides of 2 similar triangles are in the ratio 4 : 9. What is the ratio areas of these triangles.

SECTION – B

Questions 6 to 12 carry 2 marks each.

7. Points P, Q, R and S divide the line segment joining the points A(1, 2) and B(6, 7) in 5 equal parts. Find the coordinates of the points P, Q and R.

8. Find the largest number which divides 2053 and 967 and leaves a remainder of 5 and 7 respectively.

OR

4 Bells toll together at 9.00 am. They toll after 7, 8, 11 and 12 seconds respectively. How many times will they toll together again in the next 3 hours?

9. A dice is rolled twice. Find the probability that (i) 5 will not come up either time. (ii) 5 will come up exactly one time.

10. A bag contains cards numbered from 1 to 25. A card is drawn at random from the bag. Find the probability that the number on this card is (i) divisible by both 2 and 3 (ii) a two digit number

11. For what value of k, the following pair of linear equations has infinite number of solutions:
$2x+(k-2)y=k; \quad 6x+(2k-1)y=(2k+5).$

12. The sum of the first n terms of an A.P. is $5n - n^2$. Find the nth term of this A.P.

OR

Which term of the AP 21, 42, 63, 84, ... is 420?

SECTION – C

Questions 13 to 22 carry 3 marks each.

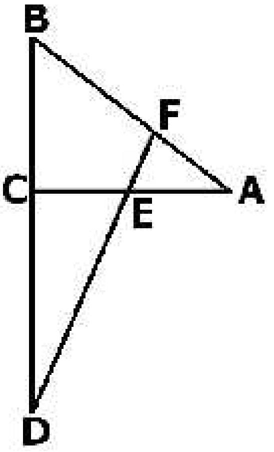
13. Prove that $7-2\sqrt{5}$ is an irrational number.

14. If the polynomial $6x^4 + 8x^3 + 17x^2 + 21x + 7$ is divided by another polynomial $3x^2 + 4x + 1$, the remainder comes out to be $(ax + b)$, find a and b.

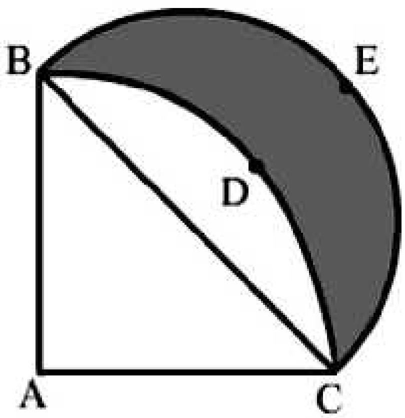
15. If AD and PM are medians of triangles ABC and PQR, respectively where $\Delta ABC \sim \Delta PQR$, prove that $\frac{AB}{PQ}=\frac{AD}{PM}$

OR

In the below figure, line segment DF intersect the side AC of a triangle ABC at the point E such that E is the mid-point of CA and $\angle AEF = \angle AFE$. Prove that $\frac{BD}{CD}=\frac{BF}{CE}$.



16. In the below figure, ABCD is a quadrant of a circle of radius 28 cm and a semi circle BEC is drawn with BC as diameter. Find the area of the shaded region. [Use $\pi = \frac{22}{7}$]



17. A quadrilateral ABDC is drawn to circumscribe a circle. Prove that AB + CD = AD + BC.

18. The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

19. Find the value(s) of p for which the points (p + 1, 2p − 2), (p −1, p) and (p − 3, 2p − 6) are collinear.

OR

The mid-point P of the line segment joining the points A(−10, 4) and B(−2, 0) lies on the line segment joining the points C(−9, −4) and D(−4, y). Find the ratio in which P divides CD. Also find the value of y.

20. A 5 m wide cloth is used to make a conical tent of base diameter 14 m and height 24 m. Find the cost of cloth used at the rate of Rs 25 per metre. [Use $\pi = \frac{22}{7}$]

OR

A girl empties a cylindrical bucket, full of sand, of base radius 18 cm and height 32 cm, on the floor to form a conical heap of sand. If the height of this conical heap is 24 cm, then find its slant height correct upto one place of decimal.

21. If A + B = 90^0, prove that $\sqrt{\frac{\tan A\tan B+\tan A\cot B}{\sin A\sec B}-\frac{\sin^2 B}{\cos^2 A}}=\tan A$

OR

Prove that: $\frac{\cos A-\sin A+1}{\cos A+\sin A-1}=\cos ecA+\cot A$.

22. Find the mode of the following frequency distribution:

Marks	Less than 20	Less than 40	Less than 60	Less than 80	Less than 100
Number of students	4	10	28	36	50

SECTION – D

Questions 23 to 30 carry 4 marks each.

23. The angle of elevation of the top of a chimney from the foot of a tower is 60° and the angle of depression of the foot of the chimney from the top of the tower is 30°. If the height of the tower is 40 m, find the height of the chimney. According to pollution control norms, the minimum height of a smoke emitting chimney should be 100 m. State if the height of the above mentioned chimney meets the pollution norms. What value is discussed in this question?

OR

A highway leads to the foot of 300 m high tower. An obeservatory is set at the top of the tower. It sees a car moving towards it at an angle of depression of 30°. After 15 seconds angle of depression becomes 60°.

(a) Find the distance travelled by the car during this time.

(b) How this observatory is helpful to regulate the traffic on the highway?

24. Construct a triangle PQR, in which PQ = 6 cm, QR = 7 cm and PR = 8 cm. Then construct another triangle whose sides are $\frac{4}{5}$ times the corresponding sides of ΔPQR.

25. If S_n denotes the sum of the first n terms of an A.P., prove that $S_{30} = 3(S_{20} - S_{10})$.

26. A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

OR

Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

27. A metallic bucket, open at the top, of height 24 cm is in the form of the frustum of a cone, the radii of whose lower and upper circular ends are 7 cm and 14 cm respectively. Find : (i) the volume of water which can completely fill the bucket. (ii) the area of the metal sheet used to make the bucket. [Use $\pi = \frac{22}{7}$]

28. Prove that : $\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \sec\theta + \tan\theta = \frac{1+\sin\theta}{\cos\theta}$

29. Prove that "If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio".

30. The following table gives production yield per hectare of wheat of 100 farms of a village.

production yield (in kg/ha)	50 - 55	55 - 60	60 - 65	65 - 70	70 - 75	75 - 80
Number of farms	2	8	12	24	38	16

Change the distribution to a more than type distribution, and draw its ogive.

OR

The median of the following data is 28. Find the values of *x* and *y*, if the total frequency is 50.

Marks	0-7	7-14	14-21	21-28	28-35	35-42	42-49
No. of Students	3	x	7	11	y	16	9

SAMPLE PAPER 04 (2018-19)

SUBJECT: MATHEMATICS **MAX. MARKS : 80**
CLASS : X **DURATION : 3 HRS**

General Instruction:

(i) **All** questions are compulsory.
(ii) This question paper contains **30** questions divided into four Sections A, B, C and D.
(iii) **Section A** comprises of 6 questions of **1 mark** each. **Section B** comprises of 6 questions of **2 marks** each. **Section C** comprises of 10 questions of **3 marks** each and **Section D** comprises of 8 questions of **4 marks** each.
(iv) There is no overall choice. However, an internal choice has been provided in two questions in 1 mark each, two questions in 2 marks each, four questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of Calculators is not permitted

SECTION – A

Questions 1 to 6 carry 1 mark each.

1. Find the common difference of the AP: $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}, ...$

2. Show that 12^n cannot end with the digit 0 or 5 for any natural number *n*.

3. Find the value of k for which the quadratic equation $4x^2 – 3kx + 1 = 0$ has two real equal roots.

OR

If $ax^2 + bx + c = 0$ has equal roots, what is the value of c?

4. If A(6, –1), B(1, 3) and C(k, 8) are three points such that AB= BC, find the value of k.

5. If $\cos – = \frac{1}{2}$, $\sin ¤ = \frac{1}{2}$ then find the value of $– + ¤$.

OR

Express cot 85° + cos 75° in terms of trigonometric ratios of angles between 0° and 45°.

6. In triangle ABC, DE || BC and $\frac{AD}{DB} = \frac{3}{5}$.If AC = 4.8 cm, find AE.

SECTION – B

Questions 6 to 12 carry 2 marks each.

7. Prove that the points (7, 10), (−2, 5) and (3, −4) are the vertices of an isosceles right triangle.

8. Find the least number which when divided by 6, 15 and 18 leave remainder 5 in each case.

OR

Find HCF and LCM of 448, 1008 and 168 using fundamental theorem of arithmetic.

9. A card is drawn at random from a well shuffled pack of 52 playing cards. Find the probability that the drawn card is (i) neither a king nor a queen (ii) red queen card.

10. A box contains 90 discs, numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears (i) a prime-number less than 23 (ii) a perfect square number.

11. Find the value of k, so that the following system of equations has no solution:
$3x + y = 1; \quad (2k-1)x + (k-1)y = (2k-1).$

12. How many three digit natural numbers are divisible by 7?

OR

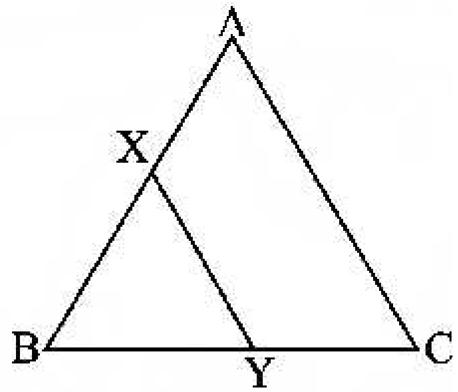
The 6th term of an Arithmetic Progression (AP) is –10 and its 10th term is –26. Determine the 15th term of the AP.

SECTION – C

Questions 13 to 22 carry 3 marks each.

13. Prove that $\sqrt{2} + \sqrt{3}$ is an irrational number.

14. In the below figure, the line segment XY is parallel to side AC of ΔABC and it divides the triangle into two equal parts of equal areas. Find the ratio $\frac{AX}{AB}$.



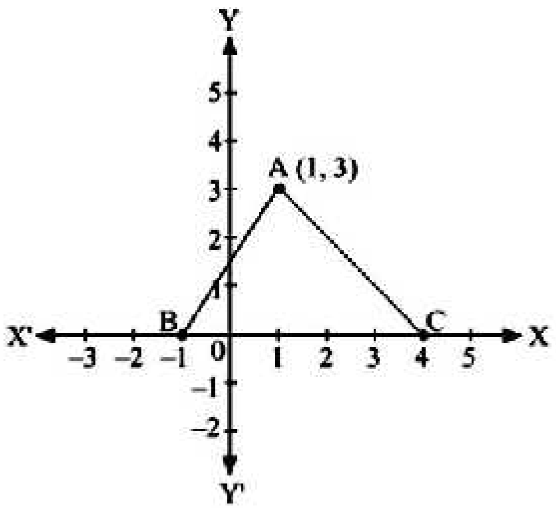
OR

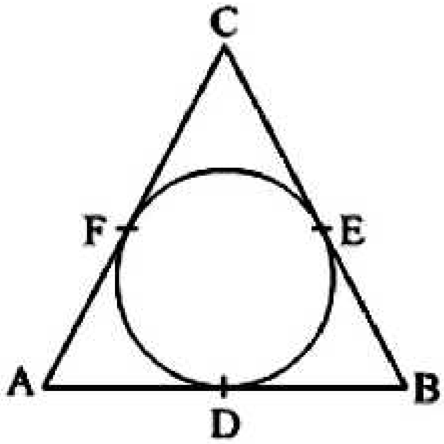
Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that ΔABC ~ ΔPQR.

15. The three vertices of a parallelogram ABCD are A(3, −4), B(−1, −3) and C(−6, 2). Find the coordinates of vertex D.

OR

In the below figure, find the area of triangle ABC (in sq. units).

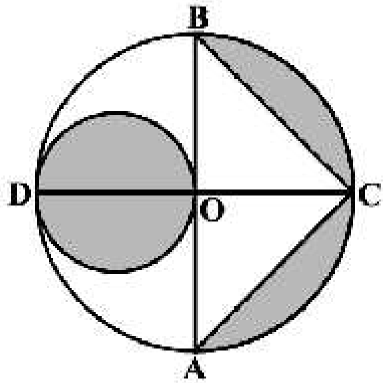




16. In the above right sided figure, a circle inscribed in triangle ABC touches its sides AB, BC and AC at points D, E and F respectively. If AB = 12 cm, BC = 8 cm and AC = 10 cm, then find the lengths of AD, BE and CF.

17. The sum of a two-digit number and the number formed by interchanging its digit is 110. If 10 is subtracted from the original number, the new number is 4 more than 5 times the sum of the digits of the original number. Find the original number.

18. In the below figure, AB and CD are two diameters of a circle with centre O, which are perpendicular to each other. OB is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region. (use π = 22/7)



19. If α and β are the zeroes of the quadratic polynomial f(x) = $3x^2 - 4x + 1$, then find a quadratic polynomial whose zeroes are $\frac{\sim^2}{\ }$ and $\frac{\ ^2}{\sim}$.

20. A vessel is in the form of hemispherical bowl surmounted by a hollow cylinder of same diameter. The diameter of the hemispherical bowl is 14 cm and the total height of the vessel is 13 cm. Find the total surface area of the vessel. (use π = 22/7)

OR

A wooden toy was made by scooping out a hemisphere of same radius from each end of a solid cylinder. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the volume of wood in the toy. (use π = 22/7)

21. Evaluate: $\frac{3\cos 55^0}{7\sin 35^0} - \frac{4(\cos 70^0 \operatorname{cos}ec 20^0)}{7(\tan 5^0 \tan 25^0 \tan 45^0 \tan 65^0 \tan 85^0)}$

OR

If x = $a\sin¯ + b\cos¯$ and y = $a\cos¯ + b\sin¯$, prove that $x^2 + y^2 = a^2 + b^2$.

22. An aircraft has 120 passenger seats. The number of seats occupied during 100 flights is given below:

No. of seats	100-104	104-108	108-112	112-116	116-120
Frequency	15	20	32	18	15

Determine the mean number of seats occupied over the flights

SECTION – D

Questions 23 to 30 carry 4 marks each.

23. A bucket open at the top, and made up of a metal sheet is in the form of a frustum of a cone. The depth of the bucket is 24 cm and the diameters of its upper and lower circular ends are 30 cm and 10 cm respectively. Find the cost of metal sheet used in it at the rate of Rs 10 per 100 cm^2. [Use π = 3.14]

24. If sec¯ = x + $\frac{1}{4x}$, Prove that sec¯ + tan¯ = 2x or $\frac{1}{2x}$.

25. A fire in a building B is reported on telephone to two fire stations P and Q, 20 km apart from each other on a straight road. P observes that the fire is at an angle of 60° to the road and Q observes that it is at an angle of 45° to the road. (a) Which station should send its team and how much will this team have to travel? (b) What according to you, are the values displayed by the teams at fire stations P and Q.

OR

A highway leads to the foot of 300 m high tower. An obeservatory is set at the top of the tower. It sees a car moving towards it at an angle of depression of 30°. After 15 seconds angle of depression becomes 60°. (a) Find the distance travelled by the car during this time. (b) How this observatory is helpful to regulate the traffic on the highway?

26. Find the number of terms of the AP $18, 15\frac{1}{2}, 13, \ldots\ldots\ldots, -49\frac{1}{2}$ and find the sum of all its terms.

27. Construct a triangle with sides 5 cm, 4 cm and 6 cm. Then construct another triangle whose sides are $\frac{2}{3}$ times the corresponding sides of first triangle.

28. A peacock is sitting on the top of a pillar, which is 9m high. From a point 27 m away from the bottom of the pillar, a snake is coming to its hole at the base of the pillar. Seeing the snake the peacock pounches on it. If their speeds are equal at what distance from the whole is the snake caught?

OR

In a class test, the sum of the marks obtained by P in mathematics and science is 28. Had he got 3 more marks in mathematics and 4 marks less in science, the product of marks obtained in the two subjects would have been 180. Find the marks obtained by him in the two subjects separately.

29. Prove that "In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

30. From the following data, draw the two types of cumulative frequency curves and determine the median from the graph.

Height(in cm)	**Frequency**
140-144	3
144-148	9
148-152	24
152-156	31
156-160	42
160-164	64
164-168	75
168-172	82
172-176	86
176-180	34

OR

Compare the modal ages of two groups of students appearing for an entrance examination:

Age(in years)	16-18	18-20	20-22	22-24	24-26
Group A	50	78	46	28	23
Group B	54	89	40	25	17

SAMPLE PAPER 05 (2018-19)

SUBJECT: MATHEMATICS | **MAX. MARKS : 80**
CLASS : X | **DURATION : 3 HRS**

General Instruction:

(i) **All** questions are compulsory.
(ii) This question paper contains **30** questions divided into four Sections A, B, C and D.
(iii) **Section A** comprises of 6 questions of **1 mark** each. **Section B** comprises of 6 questions of **2 marks** each. **Section C** comprises of 10 questions of **3 marks** each and **Section D** comprises of 8 questions of **4 marks** each.
(iv) There is no overall choice. However, an internal choice has been provided in two questions in 1 mark each, two questions in 2 marks each, four questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of Calculators is not permitted

SECTION – A

Questions 1 to 6 carry 1 mark each.

1. Find the value of y if the first three terms of an AP respectively are 3y – 1, 3y + 5 and 5y + 1.

2. If LCM (480, 672) = 3360, find HCF (480,672).

3. Find the values of k for which the quadratic equation $(k + 4)x^2 + (k + 1)x + 1 = 0$ has equal roots.

OR

Find the roots of the quadratic equation by factorisation: x2 – 9x + 20 = 0

4. If the points A(x, 2), B(−3, −4) and C(7, − 5) are collinear, then find the value of x.

5. If sin5θ = cos4θ, where 5θ and 4θ are acute angles, find the value of θ.

OR

If $\sin\theta = \frac{1}{3}$,then find the value of $(2\cot^2\theta + 2)$

6. If $\Delta ABC \sim \Delta PQR$, BC = 8 cm and QR = 6 cm, the ratio of the areas of ΔABC and ΔPQR

SECTION – B

Questions 6 to 12 carry 2 marks each.

7. Prove that the points (7, 10), (−2, 5) and (3, −4) are the vertices of an isosceles right triangle.

8. Find the HCF of 81 and 237 and express it as a linear combination of 81 and 237.

OR

Find HCF and LCM of 625, 1125 and 2125 using fundamental theorem of arithmetic.

9. If two different dice are rolled together, find the probability of getting (i) an even number on first dice (ii) an even number on both dice.

10. A bag contains cards numbered from 1 to 49. A card is drawn from the bag at random, after mixing the cards thoroughly. Find the probability that the number on the drawn card is: (i) a multiple of 5 (ii) a perfect square

11. Find the value of k, so that the following system of equations has no solution:
$(3k+1)x+3y-2=0; \quad (k^2+1)x+(k-2)y-5=0.$

12. If the seventh term of an AP is $\frac{1}{9}$ and its ninth term is $\frac{1}{7}$, find its 63rd term.

OR

If 5 times the 5th term of an AP is equal to 10 times the 10th term, show that its 15th term is zero.

SECTION – C

Questions 13 to 22 carry 3 marks each.

13. Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8.

14. ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO}=\frac{CO}{DO}$

OR

Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR. Show that ΔABC ~ ΔPQR.

15. Find the ratio in which the point P(x, 2) divides the line segment joining the points A(12, 5) and B(4, – 3). Also find the value of x.

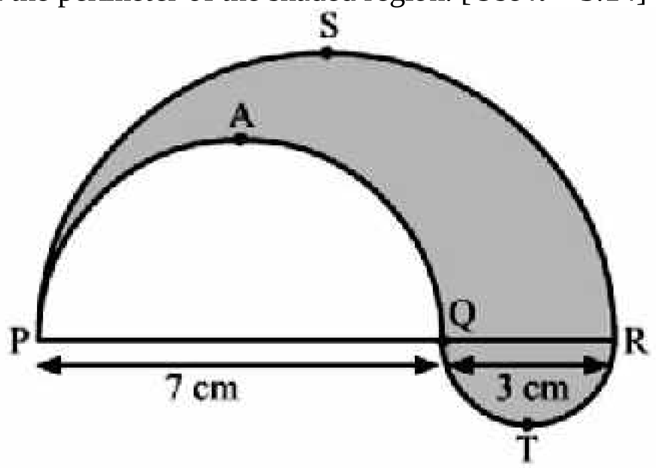
OR

If the points A(−2, 1), B(a, b) and C(4, −1) are collinear and a − b = 1, find the values of a and b.

16. If from an external point P of a circle with centre O, two tangents PQ and PR are drawn such that $\angle$QPR = 120°, prove that 2PQ = PO.

17. 8 men and 12 boys can finish a piece of work in 10 days while 6 men and 8 boys finish it in 14 days. Find the time taken by one man alone and by one boy alone to finish the work.

18. In the below figure, PSR, RTQ and PAQ are three semicircles of diameters 10 cm, 3 cm and 7 cm respectively. Find the perimeter of the shaded region. [Use π = 3.14]



19. If the polynomial $6x^4 + 8x^3 - 5x^2 + ax + b$ is exactly divisible by the polynomial $2x^2 - 5$, then find the values of a and b.

20. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank which is 10 m in diameter and 2 m deep. If the water flows through the pipe at the rate of 4 km per hour, in how much time will the tank be filled completely?

OR

A solid metallic right circular cone 20 cm high and whose vertical angle is 60°, is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter $\frac{1}{12}$ cm, find the length of the wire.

21. Evaluate: $\frac{\sin 18^0}{\cos 72^0} + \sqrt{3}\left\{\tan 10^0 \tan 30^0 \tan 40^0 \tan 50^0 \tan 80^0\right\}$

OR

If tan (A – B) = $\frac{1}{\sqrt{3}}$ and tan (A + B) = $\sqrt{3}$, then find the value of A and B.

22. Find the mode age of the patients from the following distribution :

Age(in years)	6-15	16-25	26-35	36-45	46-55	56-65
No. of patients	6	11	21	23	14	5

SECTION – D

Questions 23 to 30 carry 4 marks each.

23. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 60 m high, find the height of the building.

24. Draw a right triangle ABC in which AB = 6 cm, BC = 8 cm and $\angle$B = 90°. Draw BD perpendicular from B on AC and draw a circle passing through the points B, C and D. Construct tangents from A to this circle.

25. In an AP of 50 terms, the sum of first 10 terms is 210 and the sum of its last 15 terms is 2565. Find the A.P.

26. In a flight for 6000 km, an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 400 km/hr and consequently time of flight increased by 30 minutes. Find the original duration of flight.

OR

Out of a number of saras birds, one-fourth of the number are moving about in lots, $\frac{1}{9}$th coupled with $\frac{1}{4}$th as well as 7 times the square root of the number move on a hill, 56 birds remain in vakula trees. What is the total number of trees?

27. Sushant has a vessel, of the form of an inverted cone, open at the top, of height 11 cm and radius of top as 2.5 cm and is full of water. Metallic spherical balls each of diameter 0.5 cm are put in the vessel due to which th of the water in the vessel flows out. Find how many balls were put in the vessel. Sushant made the arrangement so that the water that flows out irrigates the flower beds. What value has been shown by Sushant?

28. If $a\cos^3 - + 3a\sin^2 - \cos - = m$ and $a\sin^3 - + 3a\sin - \cos^2 - = n$, prove that $(m+n)^{2/3} + (m-n)^{2/3} = 2a^{2/3}$

29. Prove that "In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

OR

Prove that "The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides."

30. The mean of the following frequency distribution is 57.6 and the sum of the observations is 50. Find f_1 and f_2.

Class	0-20	20-40	40-60	60-80	80-100	100-120
Frequency	7	f_1	12	f_2	8	5

OR

The following is the distribution of weights (in kg) of 40 persons:

Weight(in kg)	40-45	45-50	50-55	55-60	60-65	65-70	70-75	75-80
No. of persons	4	4	13	5	6	5	2	1

Construct a cumulative frequency distribution (of less than type) table for the data above and determine the median from the graph.

SAMPLE PAPER 06 (2018-19)

SUBJECT: MATHEMATICS **MAX. MARKS : 80**
CLASS : X **DURATION : 3 HRS**

General Instruction:

(i) **All** questions are compulsory.
(ii) This question paper contains **30** questions divided into four Sections A, B, C and D.
(iii) **Section A** comprises of 6 questions of **1 mark** each. **Section B** comprises of 6 questions of **2 marks** each. **Section C** comprises of 10 questions of **3 marks** each and **Section D** comprises of 8 questions of **4 marks** each.
(iv) There is no overall choice. However, an internal choice has been provided in two questions in 1 mark each, two questions in 2 marks each, four questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of Calculators is not permitted

SECTION – A

Questions 1 to 6 carry 1 mark each.

1. Express each of the following positive integers as the product of its prime factors: (i) 140 (ii) 156

2. For what value of p, are 2p + 1, 13, 5p – 3 three consecutive terms of an AP?

3. If $x = -\frac{1}{2}$ is a solution of the quadratic equation $3x^2 + 2kx - 3 = 0$, find the value of k.

OR

For what value of k does (k – 12)x2 + 2(k – 12)x + 2 = 0 have equal roots?

4. If the mid-point of the line segment joining the points P(6, b – 2) and Q(– 2, 4) is (2, – 3), find the value of *b*.

5. In ΔABC, right-angled at B, AB = 5 cm and $\angle$ACB = 30° then find the length of the side BC.

OR

If sin 3θ = cos (θ – 6°) here, 3θ and (θ – 6°) are acute angles, find the value of θ.

6. The areas of two similar triangles ΔABC and ΔDEF are 144 cm^2 and 81 cm^2, respectively. If the longest side of larger ΔABC be 36 cm, then find the longest side of the similar triangle ΔDEF .

SECTION – B

Questions 6 to 12 carry 2 marks each.

7. If A(5, 2), B(2, −2) and C(−2, t) are the vertices of a right angled triangle with $\angle$B = 90°, then find the value of t.

8. Two tankers contain 850 litres and 680 litres of kerosene oil respectively. Find the maximum capacity of a container which can measure the kerosene oil of both the tankers when used an exact number of times.

OR

Find the HCF and LCM of 6, 72 and 120 using fundamental theorem of arithmetic.

9. Two different dice are rolled together. Find the probability of getting : (i) the sum of numbers on two dice to be 5. (ii) even numbers on both dice.

10. A box contains 20 cards numbered from 1 to 20. A card is drawn at random from the box. Find the probability that the number on the drawn card is (i) divisible by 2 or 3 (ii) a prime number

11. Solve for x and y: $(a-b)x+(a+b)y=a^2-2ab-b^2; (a+b)(x+y)=a^2+b^2$

12. Find the middle term of the A.P. 6, 13, 20, ... , 216.

OR

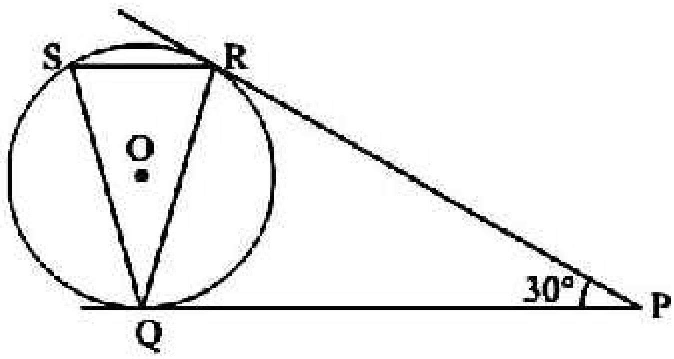
In an AP, the 24th term is twice the 10th term. Prove that the 36th term is twice the 16th term.

SECTION – C

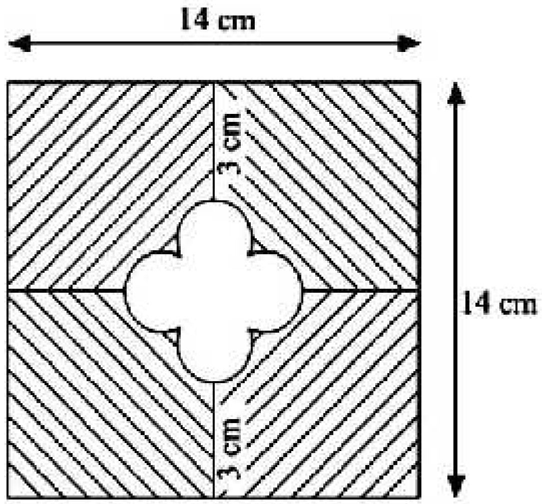
Questions 13 to 22 carry 3 marks each.

13. Show that the product of three consecutive natural numbers is divisible by 6.

14. In the below figure, tangents PQ and PR are drawn from an external point P to a circle with centre O, such that $\angle$RPQ = 30°. A chord RS is drawn parallel to the tangent PQ. Find $\angle$RQS.



15. In the below figure, find the area of the shaded region. [Use π = 3.14]



16. Find all the zeroes of the polynomial $2x^4-9x^3+5x^2+3x-1$, if two of its zeroes are $2+\sqrt{3}$ and $2-\sqrt{3}$.

17. Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.

18. BL and CM are medians of a triangle ABC right angled at A. Prove that $4(BL^2 + CM^2) = 5\ BC^2$.

OR

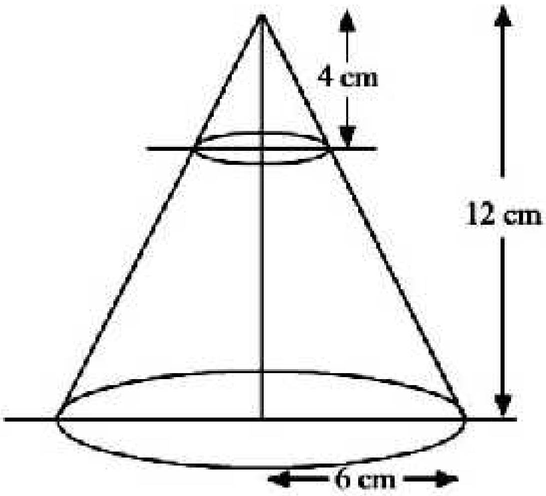
In a Δ PQR, $PR^2 - PQ^2 = QR^2$ and M is a point on side PR such that $QM \perp PR$. Prove that $QM^2 = PM \times MR$.

19. If A(−4, 8), B(−3, −4), C(0, −5) and D(5, 6) are the vertices of a quadrilateral ABCD, find its area.

OR

Find the area of the triangle ABC with A(1, −4) and mid-points of sides through A being (2, −1) and (0, −1).

20. In the below figure, from the top of a solid cone of height 12 cm and base radius 6 cm, a cone of height 4 cm is removed by a plane parallel to the base. Find the total surface area of the remaining solid.



OR

A well of diameter 4 m is dug 14 m deep. The earth taken out is spread evenly all around the well to form a 40 cm high embankment. Find the width of the embankment.

21. Evaluate: $\dfrac{\sec\theta \operatorname{cosec}(90^0 - \theta) - \tan\theta \cot(90^0 - \theta) + (\sin^2 35^0 + \sin^2 55^0)}{\tan 10^0 \tan 20^0 \tan 45^0 \tan 70^0 \tan 80^0}$

OR

If $\left(\tan\text{¯} + \sin\text{¯}\right) = m$ and $\left(\tan\text{¯} - \sin\text{¯}\right) = n$ prove that $\left(m^2 - n^2\right)^2 = 16mn$

22. Find the mode marks from the following data:

Marks	Below 10	Below 20	Below 30	Below 40	Below 50
Number of students	15	45	90	102	120

SECTION – D

Questions 23 to 30 carry 4 marks each.

23. From a point P on the ground the angle of elevation of the top of a tower is 30° and that of the top of a flag staff fixed on the top of the tower, is 60°. If the length of the flag staff is 5 m, find the height of the tower.

OR

Two poles of equal heights are standing opposite to each other on either side of the road which is 80 m wide. From a point P between them on the road, the angle of elevation of the top of a pole is 60° and the angle of depression from the top of another pole at point P is 30°. Find the heights of the poles and the distance of the point P from the poles.

24. Construct a triangle ABC with BC = 7 cm, $\angle B$ = 60° and AB = 6 cm. Construct another triangle whose sides are times the corresponding sides of ΔABC.

25. Ramkali required Rs 2,500 after 12 weeks to send her daughter to school. She saved Rs 100 in the first week and increased her weekly saving by Rs 20 every week. Find whether she will be able to send her daughter to school after 12 weeks. What value is generated in the above situation?

26. The numerator of a fraction is 3 less than its denominator. If 2 is added to both the numerator and the denominator, then the sum of the new fraction and original fraction is $\frac{29}{20}$. Find the original fraction.

OR

Solve for x : $\frac{2}{x+1}+\frac{3}{2(x-2)}=\frac{23}{5x}, x \neq 0,-1,2$

27. A solid wooden toy is in the form of a hemisphere surrounded by a cone of same radius. The radius of hemisphere is 3.5 cm and the total wood used in the making of toy is . Find the height of the toy. Also, find the cost of painting the hemispherical part of the toy at the rate of Rs 10 per cm^2.

28. Prove that: $\frac{\tan^3 \sim}{1+\tan^2 \sim}+\frac{\cot^3 \sim}{1+\cot^2 \sim}=\sec \sim \operatorname{cos}ec \sim -2\sin \sim \cos \sim$

29. Prove that “The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.”

30. Find the missing frequencies f_1 and f_2 in table given below; it is being given that the mean of the given frequency distribution is 145.

Class	100-120	120-140	140-160	160-180	180-200	**Total**
Frequency	10	f_1	f_2	15	5	**80**

OR

The following table gives the heights (in meters) of 360 trees:

Height (in m)	Less than 7	Less than 14	Less than 21	Less than 28	Less than 35	Less than 42	Less than 49	Less than 56
No. of trees	25	45	95	140	235	275	320	360

From the above data, draw an ogive and find the median

SAMPLE PAPER 07 (2018-19)

SUBJECT: MATHEMATICS | **MAX. MARKS : 80**
CLASS : X | **DURATION : 3 HRS**

General Instruction:

(i) **All** questions are compulsory.
(ii) This question paper contains **30** questions divided into four Sections A, B, C and D.
(iii) **Section A** comprises of 6 questions of **1 mark** each. **Section B** comprises of 6 questions of **2 marks** each. **Section C** comprises of 10 questions of **3 marks** each and **Section D** comprises of 8 questions of **4 marks** each.
(iv) There is no overall choice. However, an internal choice has been provided in two questions in 1 mark each, two questions in 2 marks each, four questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of Calculators is not permitted

SECTION – A

Questions 1 to 6 carry 1 mark each.

1. Find the 9th term from the end (towards the first term) of the A.P. 5, 9, 13, ..., 185.

2. State the Fundamental theorem of Arithmetic.

3. If $x = \frac{2}{3}$ and $x = -3$ are roots of the quadratic equation $ax^2 + 7x + b = 0$, find the values of a and b.

OR

If the roots of quadratic equation ax2 + bx + c = 0 are equal in magnitude but opposite in sign then find the value of b.

4. If the points A(x, 2), B(−3, −4) and C(7, − 5) are collinear, then find the value of x.

5. If cot2θ = tan4θ, where 2θ and 4θ are acute angles, find the value of sin3θ.

OR

If tan A = $\frac{5}{12}$, find the value of (sin A + cos A) . sec A.

6. ABC and BDE are two equilateral triangles such that D is the midpoint of BC. Find the ratio of the areas of triangles ABC and BDE.

SECTION – B

Questions 6 to 12 carry 2 marks each.

7. Find the ratio in which y-axis divides the line segment joining the points A(5, –6) and B(–1, –4).

8. Find the smallest 4-digit number which is divisible by 18, 24 and 32.

OR

The LCM of two numbers is 14 times their HCF. The sum of LCM and HCF is 600. If one number is 280, then find the other number.

9. Cards marked with number 3, 4, 5, ..., 50 are placed in a box and mixed thoroughly. A card is drawn at random form the box. Find the probability that the selected card bears (i) a perfect square number (ii) a number divisible by 5

10. In a single throw of a pair of different dice, what is the probability of getting (i) a prime number on each dice? (ii) a total of 9 or 11?

11. Find the value of a and b for which each of the following systems of linear equations has a infinite number of solutions: $2x+3y=7; \quad (a+b+1)x+(a+2b+2)y=4(a+b)+1.$

12. How many terms of the A.P. 18, 16, 14, ... be taken so that their sum is zero?

OR

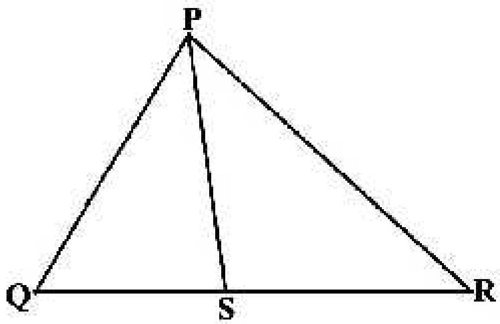
The first term, common difference and last term of an AP are 12, 6 and 252 respectively. Find the sum of all terms of this AP.

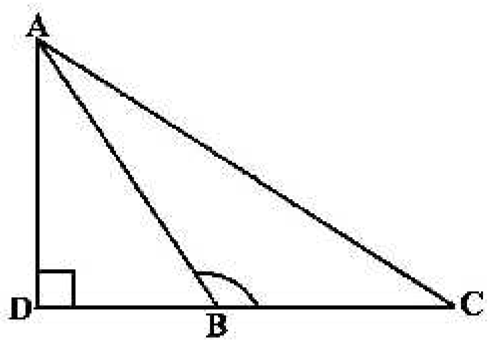
SECTION – C

Questions 13 to 22 carry 3 marks each.

13. Prove that one and only one out of n, n + 2 and n + 4 is divisible by 3, where n is any positive integer.

14. In the below left figure, PS is the bisector of $\angle$ QPR of Δ PQR. Prove that $\frac{QS}{SR} = \frac{PQ}{PR}$





OR

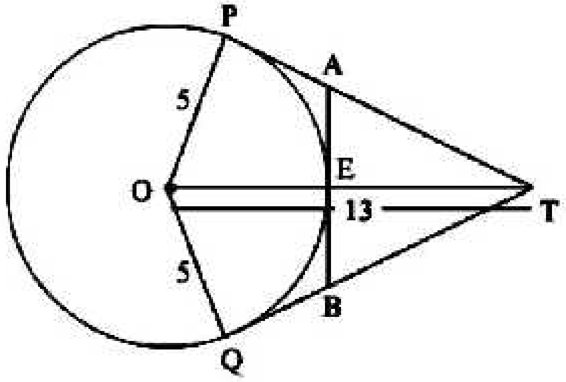
In the above right sided figure, ABC is a triangle in which $\angle$ABC > 90° and AD $\perp$ CB produced. Prove that $AC^2 = AB^2 + BC^2 + 2\ BC.BD$.

15. The x-coordinate of a point P is twice its y-coordinate. If P is equidistant from Q(2, –5) and R(–3, 6), find the coordinates of P.

OR

Prove that the area of a triangle with vertices (t, t −2), (t + 2, t + 2) and (t + 3, t) is independent of t.

16. In the below figure, O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are two tangents to the circle.



17. The present age of a woman is 3 years more than three times the age of her daughter. Three years hence, the woman's age will be 10 years more than twice the age of her daughter. Find their present ages.

18. If α and β are the zeroes of the quadratic polynomial $f(x) = 2x^2 - 5x + 7$, then find a quadratic polynomial whose zeroes are $2\sim +3$ and $2 \quad +3\sim$.

19. Find the mean marks by step deviation method from the following data:

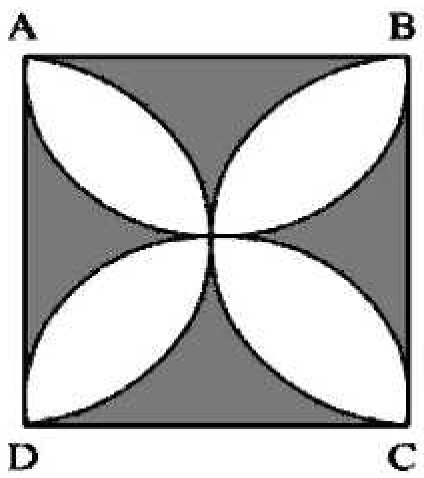
Marks	Below 10	Below 20	Below 30	Below 40	Below 50	Below 60
No. of students	4	10	18	28	40	70

20. A well of diameter 4 m is dug 21 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 3 m to form an embankment. Find the height of the embankment.

OR

The sum of the radius of base and height of a solid right circular cylinder is 37 cm. If the total surface area of the solid cylinder is 1628 sq. cm, find the volume of the cylinder.

21. In the below figure, ABCD is a square of side 14 cm. Semi-circles are drawn with each side of square as diameter. Find the area of the shaded region.



22. Evaluate: $\dfrac{\sec\theta \operatorname{cos}ec(90^0 - \theta) - \tan\theta \cot(90^0 - \theta) + (\sin^2 35^0 + \sin^2 55^0)}{\tan 10^0 \tan 20^0 \tan 45^0 \tan 70^0 \tan 80^0}$

OR

If $a^2 \sec^2\theta - b^2 \tan^2\theta = c^2$, prove that $\sin^2\theta = \dfrac{c^2 - a^2}{c^2 - b^2}$

SECTION – D

Questions 23 to 30 carry 4 marks each.

23. If the sum of first 7 terms of an A.P. is 49 and that of its first 17 terms is 289, find the sum of first n terms of the A.P.

24. A bucket open at the top is in the form of a frustum of a cone with a capacity of 12308.8 cm^3. The radii of the top and bottom circular ends are 20 cm and 12 cm, respectively. Find the height of the bucket and the area of metal sheet used in making the bucket. (use $\pi = 3.14$)

25. A bird is sitting on the top of a 80 m high tree. From a point on the ground, the angle of elevation of the bird is 45°. The bird flies away horizontally in such a way that it remained at a constant height from the ground. After 2 seconds, the angle of elevation of the bird from the same point is 30°. Find the speed of flying of the bird. (Take $\sqrt{3}$=1.732).

OR

From the top of a building 15 m high, the angle of elevation of the top of a tower is found to be 30°. From the bottom of the same building, the angle of elevation of the top of the tower is found to be 45°. Determine the height of the tower and the distance between the tower and the building.

26. Draw two concentric circles of radii 3 cm and 5 cm. Construct a tangent to smaller circle from a point on the larger circle. Also measure its length.

27. A passenger, while boarding the plane, slipped form the stairs and got hurt. The pilot took the passenger in the emergency clinic at the airport for treatment. Due to this, the plane got delayed by half an hour. To reach the destination 1500 km away in time, so that the passengers could catch the connecting flight, the speed of the plane was increased by 250 km/hour than the usual speed. Find the usual speed of the plane. What value is depicted in this question?

OR

A thief runs with a uniform speed of 100 m/minute. After one minute, a policeman runs after the thief to catch him. He goes with a speed of 100 m/minute in the first minute and increases his speed by 10 m/minute every succeeding minute. After how many minutes the policeman will catch the thief. What value is depicted in this question?

28. If $\sec^{-} + \tan^{-} = m$, show that $\left(\frac{m^2-1}{m^2+1}\right) = \sin^{-}$

29. Prove that "In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

30. If the median of the distribution given below is 14.4, find the values of *x* and *y*.

C. I.	0 – 6	6 – 12	12 – 18	18 –24	24 – 30	Total
F	4	x	5	y	1	20

OR

Draw more than ogive for the following frequency distribution:

Heights (in cms)	145-150	150-155	155-160	160-165	165-170	170-175
Number of persons	8	10	9	15	10	8

Also find the median from the graph..

SAMPLE PAPER (2018-19)

SUBJECT: MATHEMATICS(041)

BLUE PRINT : CLASS X

Unit	Chapter	VSA (1 mark)	SA – I (2 marks)	SA – II (3 marks)	LA (4 marks)	Total	Unit Total
Number system	**Real Numbers**	1(1)	**2(1)***	3(1)	--	**6(3)**	**6(3)**
Algebra	**Polynomials**	--	--	3(1)	--	**3(1)**	**20(8)**
	Pair of Linear Equations in two variables	--	2(1)	3(1)	--	**5(2)**	
	Quadratic Equations	**1(1)***	--	--	**4(1)***	**5(2)**	
	Arithmetic progression	1(1)	**2(1)**	--	4(1)	**7(3)**	
Coordinate Geometry	**Coordinate Geometry**	1(1)	2(1)	**3(1)***	--	**6(3)**	**6(3)**
Trigonometry	**Introduction to Trigonometry**	**1(1)***	--	**3(1)***	4(1)	**8(3)**	**12(4)**
	Some Applications of Trigonometry	--	--	--	**4(1)***	**4(1)**	
Geometry	**Triangles**	1(1)		**3(1)***	4(1)	**8(3)**	**15(5)**
	Circles	--	--	3(1)	--	**3(1)**	
	Constructions	--	--	--	4(1)	**4(1)**	
Mensuration	**Areas Related to Circles**	--	--	3(1)	--	**3(1)**	**10(3)**
	Surface Areas and Volumes	--	--	**3(1)***	4(1)	**7(2)**	
Statistics & probability	**Statistics**	--	--	3(1)	**4(1)***	**7(2)**	**11(4)**
	Probability	--	4(2)	--	--	**4(2)**	
	Total	**6(6)**	**12(6)**	**30(10)**	**32(8)**	**80(30)**	**80(30)**

Note: * - Internal Choice Questions

SAMPLE PAPER 08 (2018-19)

SUBJECT: MATHEMATICS **MAX. MARKS : 80**
CLASS : X **DURATION : 3 HRS**

General Instruction:
(i) **All** questions are compulsory.
(ii) This question paper contains **30** questions divided into four Sections A, B, C and D.
(iii) **Section A** comprises of 6 questions of **1 mark** each. **Section B** comprises of 6 questions of **2 marks** each. **Section C** comprises of 10 questions of **3 marks** each and **Section D** comprises of 8 questions of **4 marks** each.
(iv) There is no overall choice. However, an internal choice has been provided in two questions in 1 mark each, two questions in 2 marks each, four questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of Calculators is not permitted

SECTION – A
Questions 1 to 6 carry 1 mark each.

1. Find the 25th term of the A.P. $-5, \frac{-5}{2}, 0, \frac{5}{2}, ...$

2. State Euclid's Division Lemma.

3. Find the values of k for which the quadratic equation $(k + 4)\ x^2 + (k + 1)\ x + 1 = 0$ has equal roots.

OR

Find the discriminant of the quadratic equation: $3\sqrt{3}\,x^2 + 10x + \sqrt{3} = 0$.

4. For what values of k are the points (8, 1), (3, –2k) and (k, –5) collinear ?

5. If $\sin 3A = \cos (A – 26°)$, where 3A is an acute angle, find the value of A.

OR

If $\sec A = \frac{15}{7}$ and $A + B = 90°$, find the value of cosec B.

6. $\Delta ABC \sim \Delta DEF$ and their areas be, respectively, 64 cm^2 and 121 cm^2. If EF = 15.4 cm, find the value of BC.

SECTION – B
Questions 6 to 12 carry 2 marks each.

7. Show that the points (a, a), (–a, –a) and $(-\sqrt{3}\,a, \sqrt{3}\,a)$ are the vertices of an equilateral triangle.

8. If the HCF of 408 and 1032 is expressible in the form of 1032m – 408x5, find m.

OR

Find the largest number that divides 2053 and 967 and leaves a remainder of 5 and 7 respectively.

9. A box contains cards bearing numbers from 6 to 70. If one card is drawn at random from the box, find the probability that it bears (i) a one digit number (ii) a number divisible by 5

10. A bag contains 18 balls out of which x balls are red. (i) If one ball is drawn at random from the bag, what is the probability that it is not red? (ii) If 2 more red balls are put in the bag, the probability of drawing a red ball will be times the probability of drawing a red ball in the first case. Find the value of x.

11. For what value of k, the following pair of linear equations has infinite number of solutions: $2x + 3y = 2; \quad (k+2)x + (2k+1)y = 2(k-1).$

12. The fourth term of an A.P. is 11. The sum of the fifth and seventh terms of the A.P. is 34. Find its common difference.

OR

Find the sum of the first 25 terms of an AP whose nth term is given by an = 7 – 3n.

SECTION – C

Questions 13 to 22 carry 3 marks each.

13. Show that any positive odd integer is of the form 6q + 1 or 6q + 3 or 6q + 5 where $q \in Z$.

14. Diagonals of a trapezium ABCD with AB || CD intersects at O. If AB = 2CD, find the ratio of areas of triangles AOB and COD.

OR

Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

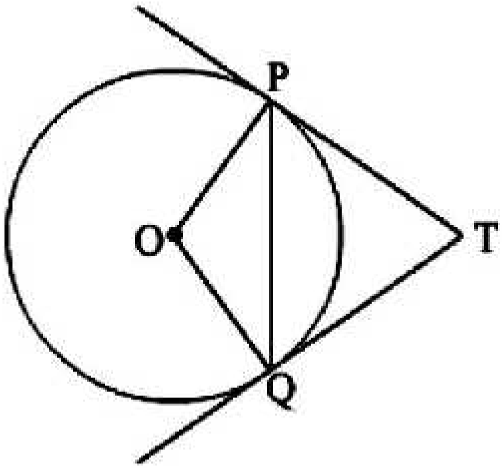
15. The base BC of an equilateral triangle ABC lies on y-axis. The coordinates of point C are (0, −3). The origin is the mid-point of the base. Find the coordinates of the points A and B.

OR

Point A lies on the line segment PQ joining P(6, –6) and Q(–4, –1) in such a way that $\frac{PA}{PQ} = \frac{2}{5}$.

If point P also lies on the line 3x + k (y + 1) = 0, find the value of k.

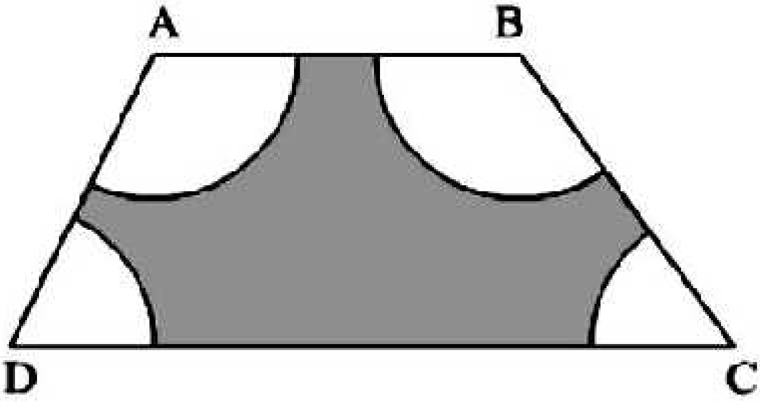
16. In the below Figure, PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the lengths of TP and TQ.



17. Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

18. Find the zeroes of the quadratic polynomial $6x^2 – 7x – 3$ and verify the relationship between the zeroes and the coefficients.

19. In the below Figure , ABCD is a trapezium with AB || DC, AB = 18 cm, DC = 32 cm and the distance between AB and DC is 14 cm. If arcs of equal radii 7 cm have been drawn, with centres A,B, C and D, then find the area of the shaded region.



20. A metallic cylinder has radius 3 cm and height 5 cm. To reduce its weight, a conical hole is drilled in the cylinder. The conical hole has a radius of $\frac{3}{2}$ cm and its depth is $\frac{8}{9}$cm. Calculate the ratio of the volume of metal left in the cylinder to the volume of metal taken out in conical shape.

OR

A solid right-circular cone of height 60 cm and radius 30 cm is dropped in a right-circular cylinder full of water of height 180 cm and radius 60 cm. Find the volume of water left in the cylinder, in cubic metres. (Use ® = $\frac{22}{7}$)

21. Evaluate: $\dfrac{\sin^2 45^0 + \frac{3}{4}\operatorname{cos}ec^2 30^0 - \cos 60^0 + \tan^2 60^0}{\sin^2 30^0 + \cos^2 60^0 + \frac{1}{2}\sec^2 45^0}$

OR

Prove that: $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \sec A + \tan A$.

22. Find the average height of maximum number of students from the following distribution:

Height(in cm)	160-162	163-165	166-168	169-171	172-174
No. of students	15	118	142	127	18

SECTION – D

Questions 23 to 30 carry 4 marks each.

23. Two poles of equal heights are standing opposite to each other on either side of the road which is 80 m wide. From a point P between them on the road, the angle of elevation of the top of a pole is 60° and the angle of depression from the top of another pole at point P is 30°. Find the heights of the poles and the distances of the point P from the poles.

OR

At a point A, 20 metres above the level of water in a lake, the angle of elevation of a cloud is 30°. The angle of depression of the reflection of the cloud in the lake, at A is 60°. Find the distance of the cloud from A.

24. Draw a circle of radius 3 cm. From a point P, 7 cm away from its centre draw two tangents to the circle. Measure the length of each tangent.

25. In an AP of 50 terms, the sum of first 10 terms is 210 and the sum of its last 15 terms is 2565. Find the A.P.

26. A fast train takes 3 hours less than a slow train for a journey of 600 km. If the speed of the slow train is 10 km/hour less than that of the fast train, find the speeds of the two trains.

OR

The side of a square exceeds the side of another square by 4 cm and the sum of the areas of the two squares is 400 sq. cm. Find the dimensions of the squares.

27. If $\cos ec\bar{\ } - \sin\bar{\ } = a^3$ and $sec\bar{\ } - \cos\bar{\ } = b^3$, prove that $a^2b^2(a^2+b^2)=1$

28. Milk in a container, which is in the form of a frustum of a cone of height 30 cm and the radii of whose lower and upper circular ends are 20 cm and 40 cm respectively, is to be distributed in a camp for flood victims. If this milk is available at the rate of Rs 35 per litre and 880 litres of milk is needed daily for a camp, find how many such containers of milk are needed for a camp and what cost will it put on the donor agency for this. What value is indicated through this by the donor agency ?

29. Prove that "The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides."

30. If the median of the distribution given below is 28.5, find the values of *x* and *y*.

C. I.	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	Total
F	5	x	20	15	y	5	100

OR

For the following distribution, draw the cumulative frequency curve more than type and hence obtain the median from the graph.

Marks	Below 10	Below 20	Below 30	Below 40	Below 50	Below 60
No. of Students	6	15	29	41	60	70

SAMPLE PAPER 09 (2018-19)

SUBJECT: MATHEMATICS | **MAX. MARKS : 80**
CLASS : X | **DURATION : 3 HRS**

General Instruction:

(i) **All** questions are compulsory.
(ii) This question paper contains **30** questions divided into four Sections A, B, C and D.
(iii) **Section A** comprises of 6 questions of **1 mark** each. **Section B** comprises of 6 questions of **2 marks** each. **Section C** comprises of 10 questions of **3 marks** each and **Section D** comprises of 8 questions of **4 marks** each.
(iv) There is no overall choice. However, an internal choice has been provided in two questions in 1 mark each, two questions in 2 marks each, four questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of Calculators is not permitted

SECTION – A

Questions 1 to 6 carry 1 mark each.

1. Show that 12^n cannot end with the digit 0 or 5 for any natural number *n*.

2. If one root of the quadratic equation $6x^2 - x - k = 0$ is $\frac{2}{3}$, then find the value of k.

OR

If two roots of $2x^2 + bx + c = 0$ are reciprocal of each other then find the value of c.

3. For what values of k are the points (8, 1), (3, –2k) and (k, –5) collinear ?

4. Find the 20th term of the A.P. $-5, \frac{-5}{2}, 0, \frac{5}{2}, ...$

5. If $\tan 9\theta = \cot\theta$ and $9\theta < 90^0$, then find the value of $\mathrm{cosec}\, 5\theta$.

OR

In ΔPQR, right angled at Q, PQ = 3 cm and PR = 6 cm, find sin R.

6. A girl walks 200 towards East and the she walks 150m towards North. Find the distance of the girl from the starting point.

SECTION – B

Questions 6 to 12 carry 2 marks each.

7. If two adjacent vertices of a parallelogram are (3, 2) and (−1, 0) and the diagonals intersect at (2, −5), then find the coordinates of the other two vertices.

8. Prove that $\sqrt{2}$ is an irrational number.

OR

Find the HCF and LCM of 96 and 404 using fundamental theorem of arithmetic.

9. Find the probability that in a leap year there will be 53 Tuesdays.

10. Two different dice are thrown together. Find the probability that the product of the numbers appeared is less than 18.

11. If seven times the 7th term of an A.P. is equal to eleven times the 11th term, then what will be its 18th term?

OR

Find the sum of all the natural numbers less than 100 which are divisible by 6.

12. Solve for x and y: $47x + 31y = 63; 31x + 47y = 15$.

SECTION – C

Questions 13 to 22 carry 3 marks each.

13. If d is the HCF of 56 and 72, find x, y satisfying d = 56x + 72y. Also show that x and y are not unique.

14. Diagonals of a trapezium PQRS intersect each other at the point O, PQ RS and PQ = 3 RS.
Find the ratio of the areas of triangles POQ and ROS.

OR

Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

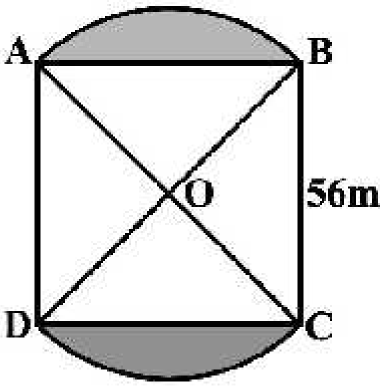
15. Show that Δ ABC with vertices A (–2, 0), B (0, 2) and C (2, 0) is similar to Δ DEF with vertices D (–4, 0), F (4, 0) and E (0, 4).

OR

Find the coordinates of the points of trisection of the line segment joining the points (3, –2) and (–3, –4).

16. Prove that the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

17. In the below figure, two circular flower beds have been shown on two sides of a square lawn ABCD of side 56 m. If the centre of each circular flower bed is the point of intersection O of the diagonals of the square lawn, find the sum of the areas of the lawn and the flower beds.



18. A solid sphere of diameter 6 cm is dropped in a right circular cylindrical vessel partly filled with water. The diameter of the cylindrical vessel is 12 cm. If the sphere is completely submerged in water, by how much will the level of water rise in the cylindrical vessel?

OR

A well of diameter 3 m is dug 14 m deep. The soil taken out of it is spread evenly all around it to a width of 5 m to form an embankment. Find the height of the embankment.

19. Draw the graphs of the equations $4x - y - 8 = 0; 2x - 3y + 6 = 0$. Also determine the vertices of the triangle formed by the lines and x-axis.

20. Evaluate: $\dfrac{\cos^2 40^0+\cos^2 50^0}{\sin^2 40^0+\sin^2 50^0}+\dfrac{4(\cos 70^0 \operatorname{cos}ec 20^0)}{7(\tan 5^0 \tan 25^0 \tan 45^0 \tan 65^0 \tan 85^0)}$

OR

Prove that: $\left(1+\dfrac{1}{\tan^2 A}\right)\left(1+\dfrac{1}{\cot^2 A}\right)=\dfrac{1}{\sin^2 A-\sin^4 A}$.

21. If ~, are the zeroes of the polynomials f(x) = x^2 – 3x + 6, then find the value of $\dfrac{1}{\sim}+\dfrac{1}{}+\sim^2+\ {}^2-2\sim$

22. The following table show the marks of 85 students of a class X in a school. Find the modal marks of the distribution:

Marks(Below)	10	20	30	40	50	60	70	80	90	100
Number of Students	5	9	17	29	45	60	70	78	83	85

SECTION – D

Questions 23 to 30 carry 4 marks each.

23. Draw a circle of radius of 3 cm. Take two points P and Q on one of its diameters extended on both sides, each at a distance of 7 cm on opposite sides of its centre. Draw tangents to the circle from these two points P and Q.

24. From the top of a hill, the angles of depression of two consecutive kilometer stones due east are found to be 45° and 30° respectively. Find the height of the hill.

OR

The angle of elevation of an aeroplane from a point A on the ground is 60°. After a flight of 30 seconds, the angle of elevation changes to 30°. If the plane is flying at a constant height of $3600\sqrt{3}$ m, find the speed in km/hr of the plane.

25. If the quadratic equation (c^2 – ab) x^2 – 2 (a^2 – bc) x + b^2 – ac = 0 in x, has equal roots, then show that either a = 0 or $a^3 + b^3 + c^3 = 3abc$.

OR

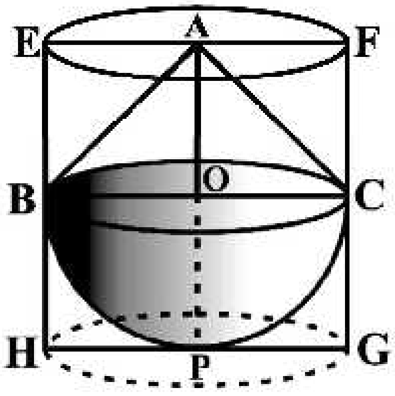
In a rectangular park of dimensions 50 m × 40 m, a rectangular pond is constructed so that the area of grass strip of uniform width surrounding the pond would be 1184 m^2. Find the length and breadth of the pond.

26. Prove that $\sqrt{\dfrac{\sec A-1}{\sec A+1}}+\sqrt{\dfrac{\sec A+1}{\sec A-1}}=2\operatorname{cos}ecA$.

27. A child puts one five-rupee coin of her saving in the piggy bank on the first day. She increases her saving by one five-rupee coin daily. If the piggy bank can hold 190 coins of five rupees in all, find the number of days she can continue to put the five-rupee coins into it and find the total money she saved. Write your views on the habit of saving.

28. Prove that “If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio”.

29. A solid toy is in the form of a hemisphere surmounted by a right circular cone (see the below figure). The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy. If a right circular cylinder circumscribes the toy, find the difference of the volumes of the cylinder and the toy. (Take $\pi = 3.14$)



30. Following is the age distribution of a group of students. Draw the cumulative frequency curve less than type and hence obtain the median from the graph.

Age(in years)	4-5	5-6	6-7	7-8	8-9	9-10	10-11	11-12	12-13	13-14	14-15	15-16
No. of students	36	42	52	60	68	84	96	82	66	48	50	16

OR

Find the missing frequencies in the following frequency distribution table, if the total frequency is 100 and median is 32.

Marks	0-10	10-20	20-30	30-40	40-50	50-60
No. of Students	10	x	25	30	y	10

SAMPLE PAPER 10 (2018-19)

SUBJECT: MATHEMATICS **MAX. MARKS : 80**
CLASS : X **DURATION : 3 HRS**

General Instruction:

(i) **All** questions are compulsory.
(ii) This question paper contains **30** questions divided into four Sections A, B, C and D.
(iii) **Section A** comprises of 6 questions of **1 mark** each. **Section B** comprises of 6 questions of **2 marks** each. **Section C** comprises of 10 questions of **3 marks** each and **Section D** comprises of 8 questions of **4 marks** each.
(iv) There is no overall choice. However, an internal choice has been provided in two questions in 1 mark each, two questions in 2 marks each, four questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.
(v) Use of Calculators is not permitted

SECTION – A

Questions 1 to 6 carry 1 mark each.

1. Find the 105th term of the A.P. $4,\ 4\frac{1}{2},\ 5,\ 5\frac{1}{2},\ 6, \ldots\ldots.$

2. Check whether 6^n can end with the digit 0 for any natural number n.

3. If 2 is a root of the equation $x^2 + bx + 12 = 0$, find the value of b.

OR

If $3x^2 - 2kx + m = 0$, find k when x = 2 and m = 3.

4. For what values of k are the points (8, 1), (3, –2k) and (k, –5) collinear ?

5. In right triangle ABC, $\angle B = 90^0$, AB = 3cm and AC = 6cm. Find $\angle C$ and $\angle A$.

OR

If $\sin\theta = x$ and $\sec\theta = y$, then find the value of $\cot\theta$.

6. If a ladder 10 m long reaches a window 8 m above the ground, then find the distance of the foot of the ladder from the base of the wall.

SECTION – B

Questions 6 to 12 carry 2 marks each.

7. Prove that the points (2, −2), (−2, 1) and (5, 2) are the vertices of a right angled triangle.

8. By using Euclids algorithm find the largest number which divides 650 and 1170.

OR

Find the HCF and LCM of 426 and 576 using fundamental theorem of arithmetic.

9. A game consist of tossing a one-rupee coin 3 times and noting the outcome each time. Ramesh will win the game if all the tosses show the same result, (i.e. either all thee heads or all three tails) and loses the game otherwise. Find the probability that Ramesh will lose the game.

10. Solve for x and y: $217x + 131y = 913;\ 131x + 217y = 827$

11. 20 tickets, on which numbers 1 to 20 are written, are mixed thoroughly and then a ticket is drawn at random out of them. Find the probability that the number on the drawn ticket is (i) a multiple of 3 or 7 (ii) a prime number.

12. If the ratio of sum of the first m and n terms of an AP is $m^2 : n^2$, show that the ratio of its mth and nth terms is $(2m - 1) : (2n - 1)$.

OR

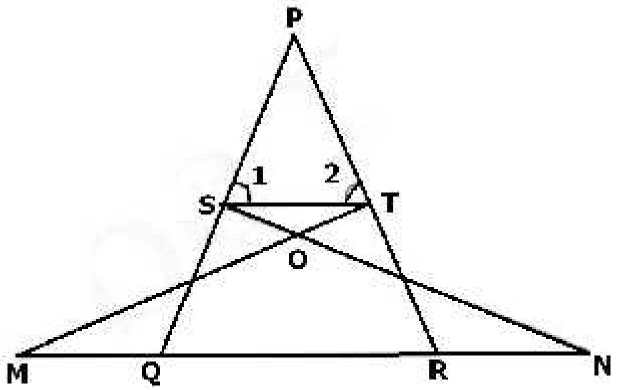
Which term of AP 3, 15, 27, 39, ... will be 120 more than its 21st term?

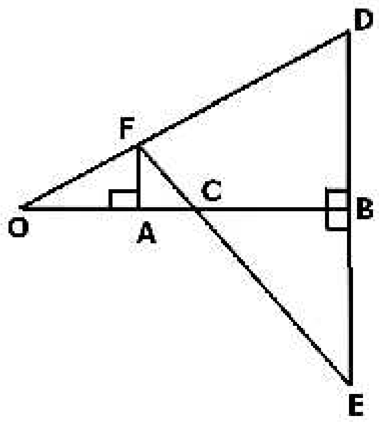
SECTION – C

Questions 13 to 22 carry 3 marks each.

13. Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

14. In the below left Figure, if $\angle 1 = \angle 2$ and $\Delta NSQ \cong \Delta MTR$, then prove that $\Delta PTS \sim \Delta PRQ$.





OR

In the above right sided Figure, OB is the perpendicular bisector of the line segment DE, FA $\perp$ OB and FE intersects OB at the point C. Prove that $\frac{1}{OA} + \frac{1}{OB} = \frac{2}{OC}$

15. The co-ordinates of the points A, B and C are (6, 3), (−3, 5) and (4, −2) respectively. P(x, y) is any point in the plane. Show that $\frac{\text{ar}(\Delta PBC)}{\text{ar}(\Delta ABC)} = \left|\frac{x + y - 2}{7}\right|$

OR

If the point C (–1, 2) divides internally the line-segment joining the points A (2, 5) and B (x, y) in the ratio 3 : 4, find the value of $x^2 + y^2$.

16. A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs 27 for a book kept for seven days, while Susy paid Rs 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

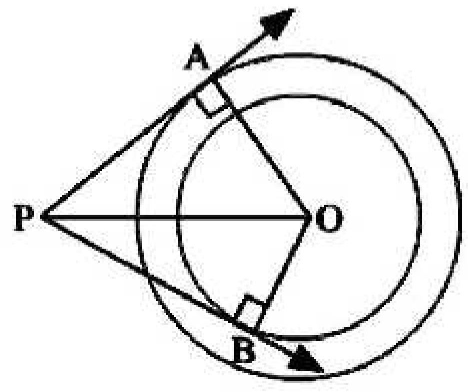
17. If the zeroes of the polynomial $2x^3 - 15x^2 + 37x - 30$ are a – b, a, a + b, find all the zeroes.

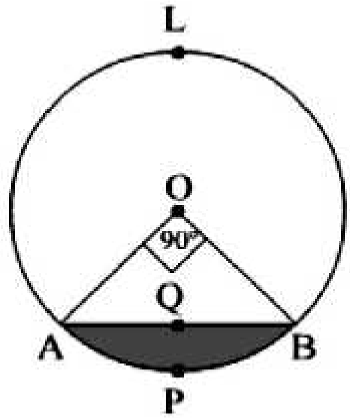
18. Evaluate: $\frac{3\cos 55^0}{7\sin 35^0} - \sqrt{3}\left\{\tan 10^0 \tan 30^0 \tan 40^0 \tan 50^0 \tan 80^0\right\}$

OR

Prove that: $(1 - \sin\theta + \cos\theta)^2 = 2(1 + \cos\theta)(1 - \sin\theta)$

19. In the below figure, there are two concentric circles of radii 6 cm and 4 cm with centre O. If AP is a tangent to the larger circle and BP to the smaller circle and length of AP is 8 cm, find the length of BP.





20. In the above right sided figure, a chord AB of a circle, with centre O and radius 10 cm subtends a right angle at the centre of the circle. Find the area of the minor segment AQBP. Hence find the area of major segment ALBQA. (use $\pi = 3.14$)

21. A cylindrical tub, whose diameter is 12 cm and height 15 cm is full of ice-cream. The whole ice-cream is to be divided into 10 children in equal ice-cream cones, with conical base surmounted by hemispherical top. If the height of conical portion is twice the diameter of base, find the diameter of conical part of ice-cream cone.

OR

A metal container, open from the top, is in the shape of a frustum of a cone of height 21 cm with radii of its lower and upper circular ends as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the container at the rate of Rs 35 per litre.

22. Find the average height of maximum number of students from the following distribution:

Height(in cm)	160-162	163-165	166-168	169-171	172-174
No. of students	15	118	142	127	18

SECTION – D

Questions 23 to 30 carry 4 marks each.

23. A vertical tower stands on a horizontal plane and is surmounted by a flagstaff of height 5 m. From a point on the ground the angles of elevation of the top and bottom of the flagstaff are 60° and 30° respectively. Find the height of the tower and the distance of the point from the tower. (Take $\sqrt{3} = 1.732$)

OR

A bird is sitting on the top of a tree, which is 80 m high. The angle of elevation of the bird, from a point on the ground is 45°. The bird flies away from the point of observation horizontally and remains at a constant height. After 2 seconds, the angle of elevation of the bird from the point of observation becomes 30°. Find the speed of flying of the bird.

24. Draw a ΔABC in which AB = 4 cm, BC = 5 cm and AC = 6 cm. Then construct another triangle whose sides are $\frac{5}{3}$ of the corresponding sides of ΔABC.

25. Reshma wanted to save at least Rs 6,500 for sending her daughter to school next year (after 12 month.) She saved Rs 450 in the first month and raised her savings by Rs 20 every next month. How much will she be able to save in next 12 months? Will she be able to send her daughter to the school next year? What value is reflected in this question.

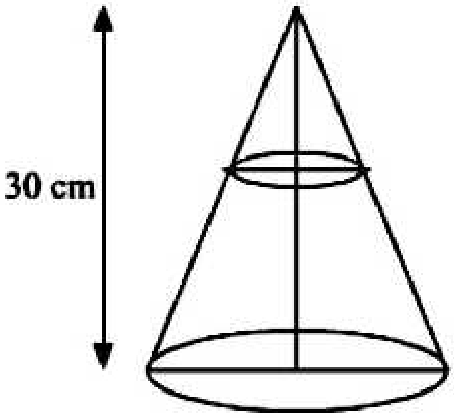
26. In a flight for 3000 km, an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 100 km/hr and consequently time of flight increased by one hour. Find the original duration of flight.

OR

A pole has to be erected at a point on the boundary of a circular park of diameter 17 m in such a way that the differences of its distances from two diametrically opposite fixed gates A and B on the boundary is 7 metres. Find the distances from the two gates where the pole is to be erected.

27. Prove that: $\frac{\sin A+\cos A}{\sin A-\cos A}+\frac{\sin A-\cos A}{\sin A+\cos A}=\frac{2}{\sin^2 A-\cos^2 A}=\frac{2}{2\sin^2 A-1}=\frac{2}{1-2\cos^2 A}$.

28. In the below figure, it is shown a right circular cone of height 30 cm. A small cone is cut off from the top by a plane parallel to the base. If the volume of the small cone is $\frac{1}{27}$ of the volume of cone, find at what height above the base is the section made.



29. Prove that "In a triangle, if the square of one side is equal to the sum of the squares of the other two sides, then the angle opposite to the first side is a right angle.

30. If the median of the distribution given below is 28.5, find the values of x and y.

C. I.	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	Total
F	5	x	20	15	y	5	100

OR

For the following distribution, draw the cumulative frequency curve more than type and hence obtain the median from the graph.

Marks	Below 10	Below 20	Below 30	Below 40	Below 50	Below 60
No. of Students	6	15	29	41	60	70